

# Non-Differentiable Object Minimum Cost System Filters Design

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**Abstract-** In this paper, we formulate the design harmonic filter problem taking by considering investment cost and operational constraints. System quality can generally be achieved by installing harmonic filters in industry systems, thereby reducing the harmonics. Users prioritize the performance and cost of filters as primary concerns. This new formulation uses a combinatorial optimization problem with a non-differentiable objective function. A solution methodology based on an optimization technique-simulated annealing is also proposed to determine the size of filters with minimum cost. As a result, the proposed solution methodology can offer the optimal size and minimum cost of design filters. In conclusion, a solution algorithm is developed to derive the global optimal solution and test on a 69kV industrial power system with highly promising results.

## **1. Introduction**

Harmonic in systems shortens the life expectancy of equipment and can interfere with communication lines and sensitive equipment. Increasing concern over this problem stems from the growing numbers and system ratings of the highly non-linear devices used in controlling apparatuses in industrial systems. The filter design has become essential for systems. The problem of designing a harmonic filter has been conventionally by trial and error approach, In the recent decade,

various formulations for a more systematic approach to harmonic filters design have been developed [1-7]. Although effective in eliminating the harmonic, these methods did not consider the cost of filter elements, they also did not address whether or not the issue of the size of filters could adhere to the system specifications.

The harmonic filters design problem has a partially discrete, partially continuous formulation with a non-differentiable nonlinear objective function. The non-differentiable nature, originating from that the cost of compensators is step-wise, makes most nonlinear optimization techniques difficult to apply. This type of problems has generally, been tackled by heuristic or approximate techniques. Consequently, those solution algorithms usually achieve local optimum rather than global optimum.

A technique employed to circumvent this problem is a technique based on Simulated Annealing (SA). SA algorithm is a powerful general-purpose technique used for solving combinatorial optimization problems. A previous study demonstrated that this algorithm asymptotically converges to the global optimal solution with the probability one [8].

In this paper, we formulate the harmonic filter design problem by taking practical aspects of the filter elements with operational constraints. Simulation results on an industry system are also presented to verify its validity.

## 2. Simulated Annealing Algorithm

This algorithm is based on the analogy between the simulation and the annealing process used for crystallization in physical systems [9]. Fig. 1 displays a pseudo code of the SA algorithm. In condensed matter physics, annealing is a thermal treatment process

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procedure SIMULATED-ANNEALING
1. Obtain an initial solution S
2. Attain an initial temperature  $T > 0$ 
3. While not yet frozen do the following
  3.1 Perform the following loop L times
    3.1.1 Generate a random neighbor  $S'$  from S
    3.1.2 If feasibility
      3.1.2.1 Let  $\Delta C = \text{cost}(S') - \text{cost}(S)$ 
      3.1.2.2 If  $\Delta C \leq 0$  (downhill move) Let  $S = S'$ 
      3.1.2.3 If  $\Delta C \geq 0$  (uphill move)
        Let  $S = S'$  with probability  $\exp(-\Delta C/T)$ 
    3.2 Let  $T = \alpha * T$  (cooling down)
4. Return S
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Fig.1. A pseudo code of the SA algorithm

which can achieve the low energy state of material. The process involves two steps: first heating up a solid to a melting point, by cooling it down until it crystallizes into a state with a perfect lattice. In the ground state, the molecular structure's arrangement has a higher structural and lowest energy.

At each temperature, perturb the present system structure  $S$  to generate a new structures  $S'$ . Then, evaluate the effect of the perturbation on the cost  $\Delta C = \text{cost}(S') - \text{cost}(S)$ , where  $\text{cost}(S)$  and  $\text{cost}(S')$  are the value of the cost function before and after the move has been executed. That is if the move decreases the value of the cost function, i.e.,  $\Delta C < 0$ . Most optimization algorithms belong to the class of greedy

search techniques. The main disadvantage associated with the greedy search technique is that it often gets stuck at local optima rather than at global optima.

However, the SA can get out of a local optimal solution in the following manner: at first, the Boltzman term,  $\exp(-\Delta C/T)$ , is calculated, where the controller parameter  $T$  is the "temperature". A random number  $\gamma$  is then selected from uniform distribution in the interval of  $[0,1]$  If  $\gamma \leq \exp(-\Delta C/T)$ , the new structure is accepted, otherwise the new move is discarded and the structure before this move is used for the next step. Due to the probabilistic selection rule, SA can always get out of a local optimal and proceed to the global optimal solution.

The feasibility checking step is used to check the new structure after a perturbation whether the constraints are satisfied or not. If all of the constraints can be satisfied, then go on next step; otherwise, the move is discarded and the structure before this move is used for next iteration.

The quality of the final solution and the speed of convergence of the SA algorithm depend on the choices of the initial temperature  $T$  in conjunction with the design of the cooling schedule. The temperature is initially set to a large value so that the probability of accepting "up-hill" move are close to 1; it is then slowly decreased

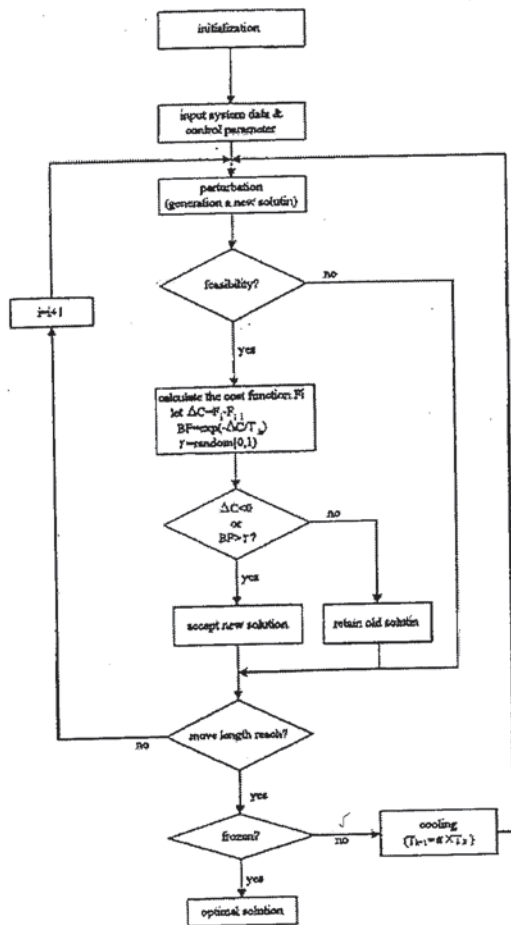


Fig. 2. The flow chart of the SA algorithm

to "frozen" according to a cooling schedule. In cooling process,  $T_{k+1} = \alpha \cdot T_k$ , where  $\alpha$  is smaller than but close to 1. Typical values lie between 0.8 to 0.99.

If the sampled mean values of cost function do not markedly change or the acceptance ration is sufficiently small (e.g., less than 1%) at five successive temperatures, then the annealing process is considered "frozen", and the global optimal structure is attained. Fig. 2 depicts a flow chart showing the major steps of the SA algorithm.

### 3. Consideration of the Design Stage

We consider the filter design problem as that identifying the size of filters with minimum cost in conjunction with operation constraints to adequately suppress harmonics.

Filters can be classified as active and passive.

Although the active filters can effectively improve the quality, it is expensive. This paper employs single tuned and double tuned passive filter structures, owing to the advantages of a simple structure, low cost and easy design.

#### A. Objective function

The objective function considered in this problem is the cost of filter:

$$F(C,L) = \sum K_{Ch} \cdot Q_{Ch} + K_{Lh} \cdot Q_{Lh} \quad (1)$$

Where C and L are the values of the capacitor and inductor of filters,  $K_{Ch}$  and  $K_{Lh}$  are the unit cost of a capacitor and an inductor, respectively. Also  $Q_{Ch}$  and  $Q_{Lh}$  are the kVA size of a capacitor and an inductor.

#### B. Constraints

The following constraints are considered:

##### (1) Environmental conditions

The frequency variation in power systems is about in  $\pm 1\%$  level.

$$-1\% < \frac{\Delta f}{f} < +1\% \quad (2)$$

The rate of capacitors has a  $\pm 2\%$  error due to the change of temperature, and has  $-5\% \sim +10\%$  error [2] due to manufactures. Therefore, the total errors due to environment and manufacture are

$$-7\% < \frac{\Delta C}{C} < +12\% \quad (3)$$

The manufacturing tolerance of inductor [3]

is about

$$-3\% < \frac{\Delta L}{L} < +3\% \quad (4)$$

The common-vibration point of the h'th order frequency is

$$n_h = \frac{1}{2\pi f \sqrt{LC}} \quad (5)$$

Where  $h=2, 3, \dots$ , express the  $h$ 'th harmonic.

From above considerations, the possible variation region of common-vibration point  $n_h$  can be express as

$$\frac{h}{1.01 \times \sqrt{1.03 \times 1.12}} < n_h < \frac{h}{0.99 \times \sqrt{0.97 \times 0.93}} \quad (6)$$

And,  $n_h$  can be summarized as

$$0.92h < n_h < 1.06h \quad (7)$$

### (2) Compensation conditions

The harmonic filters can also provide a large percentage of reactive power for power factor correction. When the capacitor,  $Q_{com}$  kVA, is installed in a system with a real power load  $P$  kW, the power factor can be improved from  $pf_0$  to  $pf_1$ , where

$$Q_{com} = P \times (\tan(\cos^{-1} pf_0) - \tan(\cos^{-1} pf_1)) \quad (8)$$

The capacitance for a single filter can be set to

$$Q_f = Q_{com} \quad (9)$$

For a multiple parallel single-tuned filter system, the capacitance corresponding to the  $h$ 'th harmonic can be distributed by

$$Q_{com} = P \times (\tan(\cos^{-1} pf_0) - \tan(\cos^{-1} pf_1)) \quad (10)$$

Where  $I_h$  is the  $h$ 'th harmonic current and  $Q_{fh}$  is the capacity of the  $h$ 'th harmonic filter. Also, the filter capacity  $Q_{fh}$  contains the capacity of capacitance  $Q_c$ , and inductor  $Q_L$ , where

$$Q_c = \frac{n_h^2}{n_h^2 - 1} Q_{fh} \quad (11)$$

$$Q_L = Q_c - Q_{fh} \quad (12)$$

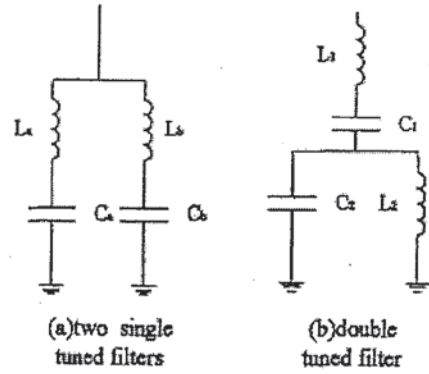


Fig.3. Element corresponding diagram

$$Q_L = \frac{1}{n_h^2} \cdot Q_c \quad (13)$$

If the reactive vars supplied by the filters exceed the system demand, the problem of system over voltage, which tends to occur at the light-load condition. For this reason, the filter capacitors are chosen such that the reactive power supplied by them does not exceed a specified value.

$$Q_{com}^{min} \leq Q_{fh} \leq Q_{com}^{max} \quad (14)$$

Where  $Q_{com}^{min}$  and  $Q_{com}^{max}$  are the minimum and maximum upper bounds on the compensation.

### (3) Operation constraint

The operational constraints can generally be consisted as follows

$$THD \leq THD_{max} \quad (15)$$

$$V_{min} \leq V_i \leq V_{max} \quad (16)$$

Where  $V_{min}$ ,  $V_{max}$  and  $THD_{max}$  correspond to the permissible minimum and maximum r.m.s. voltage, and maximum total harmonic distortion respectively.

The problem formulation of design single-tuned filters is summarized as

$$\text{Minimize } F(C,L) = K_C \cdot Q_C + K_L \cdot Q_L$$

Subject to

$$0.92h < n_h < 1.06h$$

$$Q_{fh} = \frac{1}{n_h^2} \cdot Q_{ch}$$

$$\text{THD} < \text{THD}_{\max}$$

$$V_{\min} \leq V_1 \leq V_{\max}$$

### C. Double tuned filter design

The equivalent impedances of two single tuned filters (Fig.3 (a)) near their resonance frequency are practically the same as those of a double tuned filter configuration, as illustrated in Fig. 3. (b). They have the following relationships between their components [2]:

$$C_1 = C_a + C_b \quad (17)$$

$$C_2 = \frac{C_a C_b (C_a + C_b) (L_a + L_b)^2}{(L_a C_a - L_b C_b)^2} \quad (18)$$

$$L_1 = \frac{L_a L_b}{L_a + L_b} \quad (19)$$

$$L_2 = \frac{(L_a C_a - L_b C_b)^2}{(C_a + C_b)^2 (L_a + L_b)} \quad (20)$$

Because the procedure and constraints of design a double tuned filter are similar to a single tuned filter, it will not be repeated here.

### 4. Solution Algorithm

In this section, we propose a solution algorithm for design harmonic filters to determine the size of the filters with minimum cost.

An algorithm designed as the basis of SA consists of four important elements: (1) configuration space, (2) perturbation mechanism, (3) an objective function and (4) a cooling schedule.

#### (1) Configuration space

Configuration space is the set of allowed system configurations. Design of configuration space is critical to the efficiency in iterations and the quality of the final solution. Properly designing configuration

space requires good engineering judgment.

If the upper and lower limit of filter capacity is  $Q_{com}^{\min}$  and  $Q_{com}^{\max}$ , then the solution space of  $Q_{ft}$  can be defined as

$$\{Q_{com}^{\min} \leq Q_{ft} \leq Q_{com}^{\max} \text{ specification}\}$$

#### (2) Perturbation mechanism

The goal of perturbation is to generate all possible solutions.

#### (3) Object function

The object function used in the problems of design a filter is the cost function of filters in the problem formulation.

#### (4) Cooling schedule

SA algorithm analogs to the cooling down process of material crystallize. Low speed cooling down will generate a perfect crystal; otherwise, it will fall to drawback. The cooling schedule is crucial for both the overall efficiency of iterations and the quality of final solutions. High temperature stage initially employs a high speed cooling down to improve the annealing efficiency. At a low temperature stage, it employs a low cooling schedule to improve the quality of solution.

### Solution algorithm

*Step1:* Input the system data and control parameter.

Input the system data (such as the system configuration, harmonic sources, etc.) and control parameters (e.g., the initial temperature and cooling rate)

*Step2:* Generate a feasible solution.

- (1) Randomly select a configuration from the configuration space.
- (2) Perform harmonic power flow equation to check constraints. If any constraint is violated, go to (1).

Otherwise, proceed to (3).

(3) Calculate the total cost.

Step3: Design a suitable cooling schedule.

At each temperature  $T_k$ , for  $move = 1, 2, \dots, n_k$ ,  
do step 4-6.

Step4: Generate a new feasible configuration.

- (1) Randomly select a new configuration from the configuration space.
- (2) Execute the harmonic power flow equation and check the constraints. If any constraint is violated, go to (1).
- (3) Calculate the system configuration.

Step5: Update the total cost.

Retain the new configuration or restore to the previous configuration based on the acceptance criterion (described in the section of SA algorithm).

Step6: Check the stop criterion for each temperature.

If the number of perturbations is not less than  $n_k$ , go to the next step. Otherwise, go to step4.

Step7: Check the stop criterion.

If the stop criterion is not satisfied, then the system is not yet frozen. Perform the cooling schedule, i.e. Allow  $T_{k+1} = \alpha T_k$ , then return to step 4. Otherwise, proceed to the next step.

Step8: Print out the optimal results.

## 5. Numerical Results

The numerical results in this section illustrate the proposed method's performance. The test system is a factory with a main transformer of 69/3.3kV, loading 4410kW, and power factor 0.76. There is one harmonic source. Table 1 displays the measured harmonic current, and the total harmonic distortion is 12.618%.

Three cases have been considered. In the first and second cases, a single-tuned filter was design to

reduce the 5<sup>th</sup> and 7<sup>th</sup> order harmonics respectively. Regarding the third case, a double-tuned filter was design to simultaneously reduce the 5th and 7th order harmonics. Fig.4, 5 and 6 present the simulation results for the three cases, respectively.

Moreover, Table 2 summarizes the results of SA, trial and error as well as Fermat method [9].

According to above results, the proposed SA method is better than the other two methods in terms of total harmonic distortion.

Also, the SA method can attain a minimum cost of filters.

Besides SA method, they do not use industry specification bank size for the  $Q_c$ . The cost of filters (by order) should be higher than that of SA method.

## 6. Conclusion

This paper has presented a new problem formulation for design harmonic filters in industrial power systems. The new problem formulation belongs to the non-differentiable, combinatorial optimization problem. A solution algorithm based on SA has also been developed to derive the global optimal solution and test on a 69kV industrial system with highly promising results.

Table 1: Harmonic Current

h	5	7	11	13	17	19	23	25
In	173	44	52	23	29	19	19	12

h	29	31	35	37	41	43	47	49
In	9.0	4.8	2.9	2.0	1.3	1.3	1.8	1.4

Table2: Comparison of simulation results for three methods on the third case.

	Tried and error	Fermat method	SA(single-tuned)	SA(Double-timed)
THD	3.54%	3.045%	2.6952%	2.8952%
$Q_{C5(1)}$	2200kvar	1696.06 kvar	1420 kvar	1700 kvar
$Q_{L5(1)}$	99.593 kvar	80.154 kvar	67.215 kvar	67.754 kvar
$C_{S(1)}$	488.456uf	375uf	4313.9144uf	381uf
$L_{S(1)}$	0.861mh	0.887mh	4.56543mh	0.711mh
$Q_{C7(2)}$	1800kvar	1293.24kvar	330kvar	0.1745kvar
$Q_{L7(2)}$	42.63kvar	31.1823kvar	7.9776kvar	5.8921kvar
$C_{7(2)}$	417934uf	300uf	76.5419uf	347.2uf
$L_{7(2)}$	0.550mh	0.566mh	2.22227mh	0.06mh



Fig.4.A single-tuned filter is used for the 5th order harmonics.

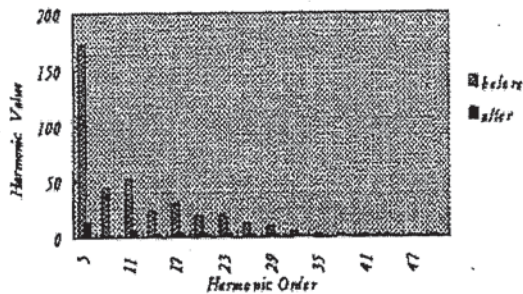


Fig. 5.A single-tuned filter is used for the 7th order harmonics.

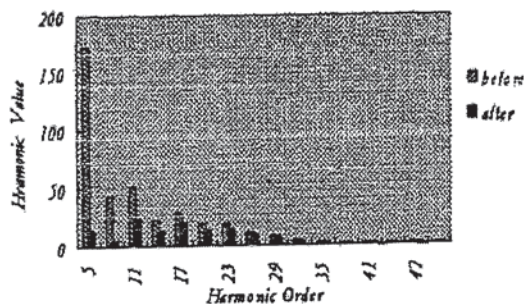


Fig. 6.Two parallel single-tuned filters are used for the 5th and 7th order harmonics.

## 7. Reference

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