A Note on "Generalized Hypercube and Hyperbus Structures for a Computer Network"

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Abstract—**This note points out that Bhuyan and Agrawal's conjectures concerning the structure of a least-cost generalized hypercube can be proved.**

Index Terms—**Generalized hypercube.**

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In paper [1], Bhuyan and Agrawal defined a *generalized hypercube* with configuration $(m_1, \cdots, m_r)$ as an undirected graph having vertex set

$$\{(x_1, \cdots, x_r): \quad 0 \le x_i < m_i, 1 \le i \le r\} \tag{5}$$

and edge set

$$\{((u_1, \cdots, u_r), (v_1, \cdots, v_r)): \\ u_i = v_i \text{ for all but one index } i, 1 \le i \le r\}. \tag{6}$$

They also defined a *nearest neighbor mesh hypercube* with configuration $(m_1, \cdots, m_r)$ to be an undirected graph with the above vertex set and having a link between two nodes $u = (u_1, \cdots, u_r)$ and $v = (v_1, \cdots, v_r)$ if and only if $\exists!\, j, 1 \le j \le r$, such that $u_j \equiv v_j + 1 (\text{mod } m_j)$ or $u_j \equiv v_j - 1 (\text{mod } m_j)$ and $u_i = v_i$ for $i \ne j$.

The generalized hypercube with configuration $(m_1, \cdots, m_r)$ has cost

$$\text{Cost}_H(m_1, \cdots, m_r) = r * \sum_{i=1}^{r} (m_i - 1). \tag{7}$$

Similarly, the nearest neighbor mesh hypercube with configuration $(m_1, \cdots, m_r)$ has cost

$$\text{Cost}_T(m_1, \cdots, m_r) = \sum_{i=1}^{r} \left\lfloor \frac{m_i}{2} \right\rfloor * (2r - k) \tag{8}$$

where $k = |\{i : m_i = 2, 1 \le i \le r\}|$.

Bhuyan and Agrawal made the following conjectures concerning optimal configurations of the above two structures.

Conjecture 1: For $N = 2^D$, a generalized hypercube has least cost if $r = \lfloor \log_4 N \rfloor$ and

$$(m_1, \cdots, m_r) = \begin{cases} (2^2, \cdots, 2^2, 2^2) & \text{if } D \text{ is even} \\ (2^2, \cdots, 2^2, 2^3) & \text{if } D \text{ is odd.} \end{cases} \tag{9}$$

Conjecture 2: For $N = 2^D$, a generalized torus has least cost if $r = \lfloor \log_8 N \rfloor$ and

$$(m_1, \cdots, m_r) = \begin{cases} (2^3, \cdots, 2^3, 2^3) & \text{if } D = 0 \bmod 3 \\ (2^2, \cdots, 2^3, 2^1) & \text{if } D = 1 \bmod 3 \\ (2^3, \cdots, 2^3, 2^2) & \text{if } D = 2 \bmod 3. \end{cases} \tag{10}$$

By this correspondence, we want to point out the following.

1) Conjecture 1 is correct for $N \ne 2^9$ and incorrect for $N = 2^9$. If $N = 2^9$, then the least-cost configuration for a generalized hypercube is $(2^3, 2^3, 2^3)$, rather than the conjectured $(2^2, 2^2, 2^2, 2^3)$.

2) Conjecture 2 is true for all $N = 2^D$.

3) Results similar to the above conjectures can be established when N is a power of a prime number.

The reader is referred to [2] for detailed discussions.

References

[1] L.N. Bhuyan and D.P. Agrawal, "Generalized hypercube and hyperbus structures for a computer network," *IEEE Trans. Comput.*, vol. C-33, no. 4, pp. 323–333, 1984.

[2] Guan I. Chen, T.H. Lai, and Y.-N. Lien, "A note on 'Generalized hypercube and hyperbus structures for a computer network'," Tech. Rep. Dep. Comput. Inform. Sci., The Ohio State Univ., Columbus, OH, 1989.