

Optimal Twin-Level Output Feedback Regulator Design

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Abstract— The purpose of this paper is to address the design of decoupled-level large system controllers using an optimal reduced order model whose state variables are all output states. The reduced-order model retains their physical meaning and is used to design a decoupled-level linear feedback controller that takes into account the realities and constraints of the large systems. The decoupled-level control strategy is used and a global control signal is generated from the output variables to minimize the effect of interactions. As a result, the effectiveness of this controller is evaluated and considerable savings in computer memory are achieved. In conclusion, the controllers determined at the subsystem level depend only on local information operating to the particular machine. Example is given to illustrate the advantages of the proposed method. Responses of the system with decoupled-level scheme and optimal reduced order scheme are included for comparative analyses. It is recommended that the proposed method can be applied to supper large system

1. Introduction

The design of large system , can be formulated as an optimal linear regulator control problem whose solution is a complete state

control scheme [1]. Thus, the implementation requires the design of state estimators [2]. These increase the hardware cost and reduce the reliability of the control system. These are the reasons that a control scheme uses only some desired state variables such as output states. Upon this, a scheme referred to as suboptimal control is obtained but only some state variables are used in the implemented control scheme while the others are omitted for convenience [3].

Obviously, this approach is arbitrary and cannot be accepted on faith. Performance degradation is not evaluated for general system sunder different conditions. The recent approach using optimal reduced order model is obtained [4] [5]. However, the optimal control strategy is also used for the reduced order model of the large system, the computation of an optimal controller becomes extremely difficult and time consuming as the order of the system increases. For an n th-order system it is necessary to solve an $n(n+1)/2$ Riccati equations in order to calculate the controller gain. And the problem formulation itself is not straight forward as it is complex to determine the design parameters in the performance criterion as the order of the system increases. To overcome these difficulties, the former paper concerned with the development of multi-level optimal stabilization of

interconnected power system in ref. [6] is applied to the proposed approach. The overall system is decomposed into separate subsystems, each subsystem comprising one machine. At the subsystem level, an optimal feedback controller is derived by output feedback of each machine. The order of this model is obviously lower than that of the overall system and the method proposed in ref [6]. Consequently, considerable savings in computer memory are achieved. The controllers thus determined at the subsystem level depend only on local information operating to the particular machine. And, since only the output feedback is used via optimal reduced order model, the control strategy can be implemented easily.

In order to take into account the interaction between the different subsystems, a global controller is designed at a higher level [7]. At this level, all subsystems will transfer the necessary information to achieve the global objectives. In this paper, the global gain is obtained from the optimal reduced order model of the whole system by using only output feedback. The evaluation of the global gain is much easier than the overall system optimal state feedback gains because the optimal reduced order model is used.

The control strategy proposed here is applied to a, two-machine system. The results of the study are presented to demonstrate the effectiveness of the two-level optimal output feedback controller. A comparison between the performance of the proposed controller and that of the optimal reduced order method and the two-level control strategy is also included.

The attractive features of the two-level optimal output feedback stabilizers design are as follows:

(1) The output state variables are some desired or available variables, thus, the state variables of the reduced order model retain their physical meaning.

(2) Local controllers determined depend only on local output information pertaining to the subsystem. Consequently, a considerable savings in computation effort at the (machine) subsystem level is achieved and no estimator is needed.

(3) Interaction between the different subsystem is minimized by the use of the global controller gain at a higher level using the output state variables of the overall system via optimal reduced order model. The evaluation process is much easier than the optimal control strategy and the transient response is much better also.

As a matter of fact this paper is an extension of optimal reduced order method proposed by Ali Feliachi et. al [4][5] and the two-level optimal stabilization method proposed by Y.L. Abdel-Magid and Gama1 M. Aly [6].

2. Background

2.1 Optimal Reduced Order Model

The linearized model of the electrical power system can be described by the following state space representation:

$$\dot{\bar{X}} = \bar{A}\bar{X} + \bar{B}u \quad (1)$$

Where \bar{X} $n \times 1$ state vector \bar{A} , \bar{B} constant matrices of appropriate dimensions Since the reduced order model derived in refs. [4] [5] is used in the following study, the process of evaluating the reduced order model is abbreviated as follows without proof.

The reduced order model is derived using the following system whose first m variables are

the desired variables z , which are speeds and torque angles in the proposed approach:

The similarity transformation T is obtained in ref. [4].

$$\dot{\hat{X}} = \hat{A} \hat{X} + \hat{B} u \quad (2)$$

$$Z = [Im, 0] \hat{X} \quad (3)$$

where

$$\hat{X} = T \bar{X}$$

$$\hat{A} = T \bar{A} T^{-1}$$

$$\hat{B} = T \bar{B}$$

$Im = m \times m$ identity matrix

Assume that the eigenvalues, of \hat{A} are distinct, this will actually be the case in the power system.

Let $V = [V_1, V_2, \dots, V_n]$ where V_i is the right eigenvector of A associated with λ_i . Let $W = V^{-1}$

$$\text{Define } \phi = W \hat{X}$$

Then:

$$\dot{\hat{\phi}} = \Lambda \phi + \Gamma u \quad (4)$$

$$Z = D \phi \quad (5)$$

where

$$\Lambda = W \hat{A} V = \text{diagonal} (\lambda_1, \lambda_2, \dots, \lambda_n)$$

$$\Gamma = W \hat{B}$$

$$D = [Im, 0] V$$

These equations can be re-arranged and written in partition form as:

$$\dot{\phi}_1 = \Lambda_1 \phi_1 + \Gamma_1 u \quad (6)$$

$$\dot{\phi}_2 = \Lambda_2 \phi_2 + \Gamma_2 u \quad (7)$$

$$Z = D_1 \phi_1 + D_2 \phi_2 \quad (8)$$

where

Λ_1 contains modes to be retained

Λ_2 contains modes to be eliminated

Assume the reduced order system we are sought to determine will be of the form as follows:

$$\dot{Z} = F Z + G u \quad (9)$$

The evaluation algorithm of F and G proposed in ref. [4] are abbreviated as follows:

$$F = D_1 \Lambda_{-1} D_1^{-1} \quad (10)$$

Let V_m be the modal matrix associated with eqn. (10) Define

$$\bar{F} = V_m^{-1} F V_m = \text{diagonal}(\lambda_1, \lambda_2, \dots, \lambda_m) \quad (11)$$

$$\bar{G} = V_m^{-1} G \quad (12)$$

$$\bar{C} = V_m^{-1} D_2 \quad (13)$$

$$\bar{\Gamma}_1 = V_m^{-1} D_1 \Gamma_1 \quad (14)$$

$$\text{Then } \bar{S} = \bar{C} \Lambda_2 - \bar{F} \bar{C} \quad (15)$$

$$\Lambda = -(\bar{F} + \bar{F}^T)^{-1} \quad (16)$$

$$R = -\Lambda \bar{S} \quad (17)$$

Let $\alpha_i = \lambda_m + i$, $i = 1, 2, \dots, n-m$

Then $\Lambda_2 = \text{diagonal} (\alpha_1, \alpha_1, \dots, \alpha_{n-m})$

The $(i, j)^{th}$ element of the $m \times p$ matrix is given by:

$$\Omega_{ij} = \frac{R_{ij}}{\lambda_i^* + \alpha_j} \quad (18)$$

where the subscript * denotes complex conjugate

$$\Delta = \Lambda^{-1} \Omega \quad (19)$$

$$\text{Let } \bar{K} = \bar{\Gamma}_1 + \bar{C} \Gamma_2 \quad (20)$$

$$\text{Then } \bar{G} = \bar{K} + \Delta \Gamma_2 \quad (21)$$

$$\text{And } G = V_m \bar{G} \quad (22)$$

2.2 Decoupled-level Optimal Stabilization

The method proposed in ref. [6] is abbreviated as follows:

A multi-machine interconnected system S can be described by a linear model of the form

$$S: \dot{X} = A X + B u \quad (23)$$

Where X is an n -dimensional state vector and U is an m -dimensional control vector. A and B are constant matrices of appropriate dimensions. The system in eqn. (23) can be considered to be composed of N interconnected subsystems, each subsystem S_i , being described

as S_i :

$$\dot{\bar{X}}_i = A_{ii} X_i + B_{ii} U_i + h_i(x), \quad i=1,2,\dots,N \quad (24)$$

such that

$$X = [X_1^T, X_2^T, \dots, X_N^T]^T \quad (25)$$

and

$$h_i(x) = \sum_{\substack{j=1 \\ j \neq i}}^N A_{ij} X_j$$

The two-level control strategy will be of the form

$$u_i = u_i^{\ell} + u_i^g \quad (27)$$

u_i^{ℓ} is a local feedback control vector assuming no interactions between subsystems, i.e., $h_i(x) = 0$. u_i^g represents a global control signal that compensates for the effect of the presence of coupling.

The global signal u^g is determined such that

$$B u^g + C X = 0 \quad (28)$$

where

$$C_{ij} = A_{ij}, \quad i \neq j \quad (29)$$

$$= 0 \quad i = j$$

And $u^g = -B' C X \quad (30)$

where B' is the pseudo-inverse of B , defined as

$$B' = [B^T B]^{-1} B^T$$

Thus

$$u^g = -[B^T B]^{-1} B^T C X = -G X \quad (31)$$

where $G = B' C = [B^T B]^{-1} B^T C$ is so called the global gain matrix

3. Control Structure

In order to stabilize the overall system S , the two-level control strategy of the form is shown in eqn. (27).

The local optimal feedback control u_i^{ℓ} can

be determined as follows:

Assuming zero interactions at the local level, then

$$\dot{\bar{X}}_i = A_{ii} X_i + B_{ii} u_i^{\ell}, \quad i=1,2,\dots,N \quad (32)$$

represents N decoupled subsystems.

These equations can be re-arranged and written in the form as:

$$\dot{\bar{X}}_i = A_{ii} \bar{X}_i + \bar{B}_{ii} u_i^{\ell}, \quad i=1,2,\dots,N \quad (33)$$

With

$$\bar{X}_i = [\bar{X}_{ir}^T, \bar{X}_{id}^T]^T$$

where

\bar{X}_{ir} modes to be retained

\bar{X}_{id} modes to be deleted

Using the expressions given in the section 2, a reduced order model of subsystem i is obtained

$$\dot{\bar{X}}_{ir} = F_i \bar{X}_{ir} + G_i u_i^{\ell}, \quad i=1,2,\dots,N \quad (34)$$

The performance of each subsystem is measured when the quadratic cost

$$J_i = \frac{1}{2} \int_0^{\infty} (\bar{X}_{ir}^T Q_i \bar{X}_{ir} + u_i^{\ell T} R_i u_i^{\ell}) dt \quad (35)$$

attains its minimum value when an optimal control u_i^{ℓ} is applied to each subsystem. Q_i and R_i are symmetric positive semi-definite and positive definite matrices, respectively.

The optimal u_i^{ℓ} minimizing eqn.(35) can be determined as

$$u_i^{\ell} = -K_i \bar{X}_{ir} \quad (36)$$

$$K_i = R_i^{-1} G_i P_i \quad (37)$$

where P_i is the solution of the Riccati equation:

$$P_i F_i + F_i^T P_i - P_i G_i R_i^{-1} G_i^T P_i + Q_i = 0 \quad (38)$$

A multi-machine interconnected system S shown in eqn.(23) can be re-arranged and written in the form as:

$$\dot{\bar{X}}' = A' X' + B' u \quad (39)$$

With

$$X' = [X_r^T, X_d^T]^T$$

where

$$X_r' = [\bar{X}_{1r}^T, \bar{X}_{2r}^T, \dots, \bar{X}_{Nr}^T]^T$$

$$\bar{X}_d' = [\bar{X}_{1d}^T, \bar{X}_{2d}^T, \dots, \bar{X}_{Nd}^T]^T$$

Using the expressions given in section 2. a reduced order model of the whole system S is obtained as

$$\dot{\bar{X}}_r = F' X_r' + G' u \quad (40)$$

The global signal u_g is determined such that

$$G' u_g + C' X_r' = 0$$

$$\text{where } C'_{ij} = F'_{ij} \quad i \neq j$$

$$= 0 \quad I = j$$

(41)

From eqn.(31), we get the global gain matrix G as

$$\hat{G} = [G'^T G']^{-1} G'^T C'$$

The overall control strategy can be shown in Fig.1

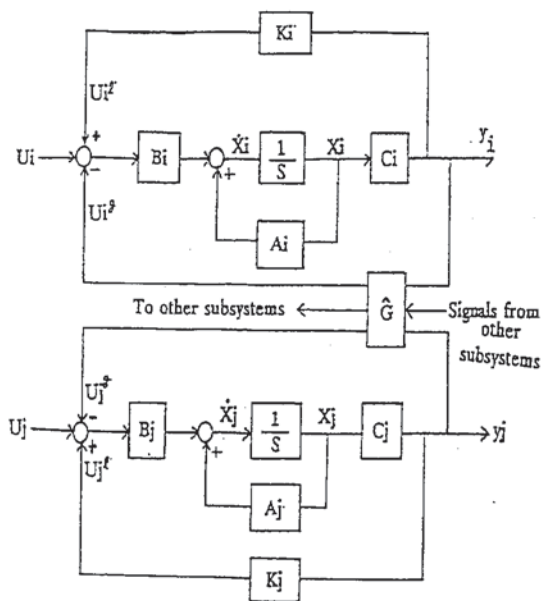


Fig.1 Decoupled-level optimal control strategy via output feedback

4. System Study

The model given in [6] is

$$\dot{X} = A X + B u \quad (43)$$

Where

$$X^T = [\Delta W_1, \Delta \delta_1, \Delta e_{q1}, \Delta V_{P1}, \Delta W_2, \Delta \delta_2, \Delta e_{q2}, \Delta V_{P2}]$$

$$A = \begin{bmatrix} -0.244 & -0.0747 & -0.1431 & 0 & 0 & 0.0747 & 0.0041 & 0 \\ 377 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.046 & -0.455 & 0.244 & 0 & 0.046 & 0.13 & 0 \\ 0 & -398.56 & -19498.8 & -50 & 0 & 398.58 & -3967 & 0 \\ 0 & 0.178 & -0.0433 & 0 & -0.2473 & -0.178 & -0.146 & 0 \\ 0 & 0 & 0 & 0 & 376.99 & 0 & 0 & 0 \\ 0 & 0.056 & 0.1234 & 0 & 0 & -0.0565 & -0.3061 & 0.149 \\ 0 & -677.39 & -10234.22 & 0 & 0 & 677.78 & -13364.16 & -50 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 25000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25000 \end{bmatrix}^T$$

Simulation results indicated that the system response is highly oscillatory. To improve the system damping using the two-level scheme, the system is decomposed as follows:

Machine (1):

$$X_1 = [\Delta \omega_1, \Delta \delta_1, \Delta e_{q1}, \Delta V_{F1}]^T$$

Machine (2):

$$X_2 = [\Delta \omega_2, \Delta \delta_2, \Delta e_{q2}, \Delta V_{F2}]^T$$

The decomposed system and control matrices are:

For system 1:

$$A = \begin{bmatrix} -0.244 & -0.0747 & -0.1431 & 0 \\ 377 & 0 & 0 & 0 \\ 0 & -0.046 & -0.455 & 0.244 \\ 0 & -398.56 & -19498.8 & -50.0 \end{bmatrix}$$

$$B = [0, 0, 0, 25000]^T$$

The eigenvalues of A_1 are $-0.127 \pm j5.206$

and $-25.22 \pm j64.38$.

Obviously, the modes that should be retained are the mechanical modes $-0.127 \pm j5.206$. Using the expressions given in section 2.1, a reduced order model is obtained.

$$\dot{\bar{X}}_1 = F_1 \bar{X}_1 + G_1 u_1 \quad (44)$$

where .

$$\bar{X}_1 = [\Delta \omega_1, \Delta \delta_1]^T$$

$$F_1 = \begin{bmatrix} -0.26 & -0.07 \\ 377 & 0 \end{bmatrix}$$

$$G_1^T = [-0.1826, -0.734]$$

For the comparative reason, the state weighting matrix Q_1 and control weighting matrix R_1 are chosen as that used in refs. [4][5] and [6].

$$Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}, R_1 = 1$$

The values of the feedback control gains are calculated as follows:

$$K_1 = [-49.3107 \quad -0.725]$$

For system 2:

$$A = \begin{bmatrix} -0.2473 & -0.178 & -0.146 & 0 \\ 377 & 0 & 0 & 0 \\ 0 & -0.0565 & -0.3061 & 0.1492 \\ 0 & 677.78 & -133641 & -50.0 \end{bmatrix}$$

$$B = [0 \quad 0 \quad 0 \quad 25000]^T$$

The eigenvalues of A_2 are $-0.988 \pm j8.35$ and $-25.19 \pm j37.13$

Obviously the modes that should be retained are the mechanical modes $-0.988 \pm j8.35$. Using the expressions given in section 2.1 a reduced order model is obtained.

$$\dot{\bar{X}}_2 = F_2 \bar{X}_2 + G_2 u_2 \quad (45)$$

where

$$\bar{X}_2 = [\Delta \omega_2, \Delta \delta_2]^T$$

$$F_2 = \begin{bmatrix} -0.18 & -0.18 \\ 377 & 0 \end{bmatrix}$$

$$G_2^T = [-0.2668 \quad -2.6202]$$

For the comparative reason, the state weighting matrix Q_2 and control weighting matrix R_2 are chosen as that used in refs. [4][5] and [6].

$$Q_2 = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}, R_2 = 1$$

The values of the feedback gains are calculated as follows:

$$K_2 = [-32.8413, -0.6864]$$

From equation (39), the interconnected system S can be rearranged and written as:

$$\dot{X} = A'X + B'u \quad (46)$$

where

$$A' = \begin{bmatrix} -0.244 & -0.0747 & 0 & 0.0749 & -0.1431 & 0 & 0.0041 & 0 \\ 377 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.178 & -0.2473 & -0.178 & -0.0433 & 0 & -0.146 & 0 \\ 0 & 0 & 376.99 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.46 & 0 & 0.046 & -0.455 & 0.244 & 0.13 & 0 \\ 0 & -398.56 & 0 & 678.58 & -19498.8 & -50 & -3967 & 0 \\ 0 & 0.056 & 0 & -0.0565 & 0.1234 & 0 & -0.3061 & 0.149 \\ 0 & -677.39 & 0 & 677.78 & -10234.22 & 0 & -13364.16 & -50 \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 25000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25000 \end{bmatrix}^T$$

$$X' = [\Delta \omega_1, \Delta \delta_1, \Delta \omega_2, \Delta \delta_2, \Delta e_{q1}, \Delta V_{F1}, \Delta e_{q2}, \Delta V_{F2}]^T$$

The eigenvalues of the original system are given in Table I

Table I SYSTEM EIGENVALUES

$-0.09 \pm j9.84$	$-25.17 \pm j67.8$
-0.0017	$-25.24 \pm j30.31$
-0.243	

The modes that should be retained are the

modes $-0.09 \pm j9.84, -0.0017, -0.243$. Using the

optimal reduced order method, the following reduced order model is obtained with the form of eqn.(40):

$$\dot{X}_r = F'X_r + G'u \quad (47)$$

Where

$$F' = \begin{bmatrix} -0.237 & -0.07 & -0.007 & 0.073 \\ 377 & 0 & 0 & 0 \\ -0.059 & 0.18 & -0.188 & -0.184 \\ 0 & 0 & 377 & 0 \end{bmatrix}$$

$$G' = \begin{bmatrix} -0.022 & -0.15 & 0.009 & 0.23 \\ 0.007 & 0.11 & -0.029 & -0.35 \end{bmatrix}^T$$

$$X_r^T = [\Delta\omega_1, \Delta\delta_1, \Delta\omega_2, \Delta\delta_2]^T$$

By ref.[8], it can be checked that the assumption of weakly coupled subsystems is not satisfied in this case and the global signal needs to be calculated.

The global control matrix G is evaluated from eqn.(42) and given by

$$G = \begin{bmatrix} 0.118 & -0.36 & 0.02 & -0.209 \\ 0.0975 & -0.2974 & 0.0141 & -0.1465 \end{bmatrix}$$

After the interconnected system level control, the interconnected system S in eqn.(43) can be written as follows:

$$\dot{X} = AX + Bu \quad (48)$$

Where

$$X^T = [\Delta\omega_1, \Delta\delta_1, \Delta e_{q1}, \Delta V_{F1}, \Delta\omega_2, \Delta\delta_2, \Delta e_{q2}, \Delta V_{F2}]$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 377 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2950 & 8601 & -19499 & -50 & -500 & 5624 & -3967 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

It is shown from eqn.(48) that the interaction between subsystems should be minimized by using the output states feedback control only.

The transient responses of the angular frequencies with and without global control to a 5% change in the mechanical torque of machine 1 are shown in Figure 2.

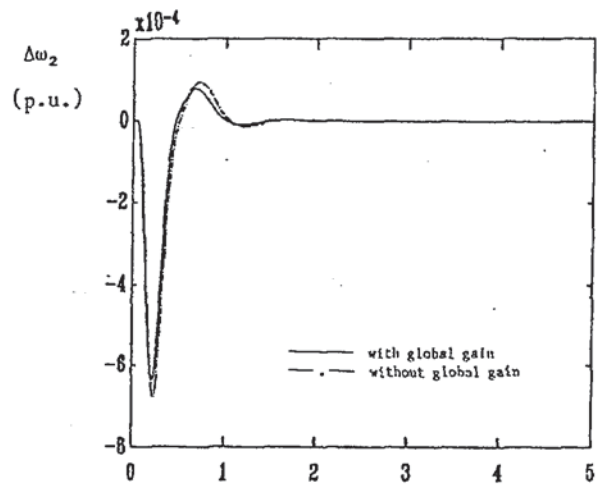
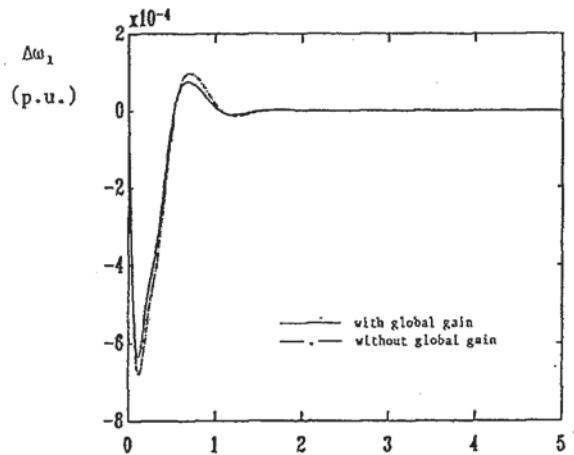


Fig.2 Transient responses of the angular frequencies with and without global control to a 5% change in the mechanical torque of machine 1.

The overall system eigenvalues are given in Table II. and the results in refs. [4][5] [6] are listed for comparative analyses.

Table II SYSTEM EIGENVALUES

Optimal [1]	Optimal Reduced Order[4] [5]	Two-level Stabilization[6]	Proposed Method
$-0.74 \pm j9.88$	$-0.6 \pm j10.2$	$-0.0305 \pm j0.1356$	$-3.8171 \pm j8.5218$
$-1.91 \pm j1.85$	$-1.2 \pm j1.91$	$-0.0744 \pm j0.1010$	$-8.5314 \pm j18.3696$
$-25.17 \pm j6.78$	$-23.3 \pm j6.72$	-0.0926	$-16.9549 \pm j19.9542$
$-25.24 \pm j30.31$	$-2.49 \pm j29.8$	-5.8469	$-21.3228 \pm j66.0701$

It is shown from Table II. That the relative stability of the proposed method is much better than others.

The transient responses of the angular frequencies to a 5% change in the mechanical torque of machine 1 and machine 2 are shown in Figure 3 and Figure 4 respectively .

The transient responses following a 5% change in the mechanical torque of both machines at the same time are shown in Figure 5.

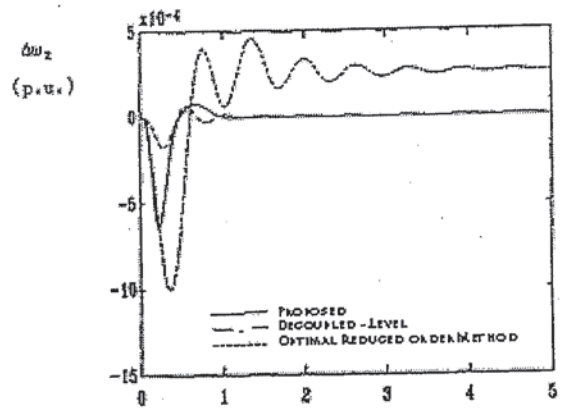
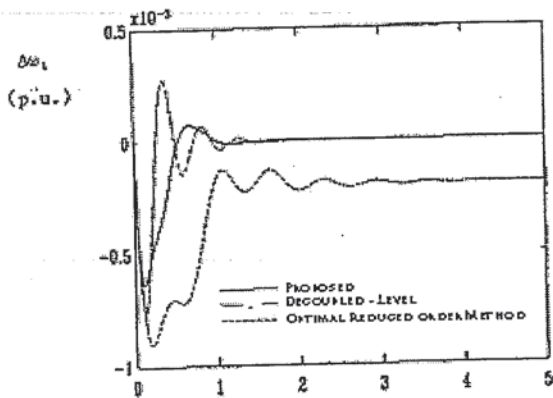


Fig. 3 Transient response following a 5% change in mechanical torque of machine 1 .

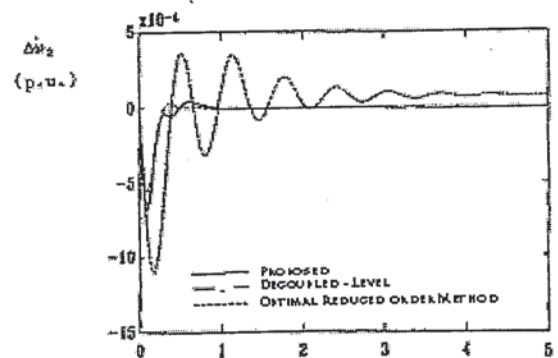
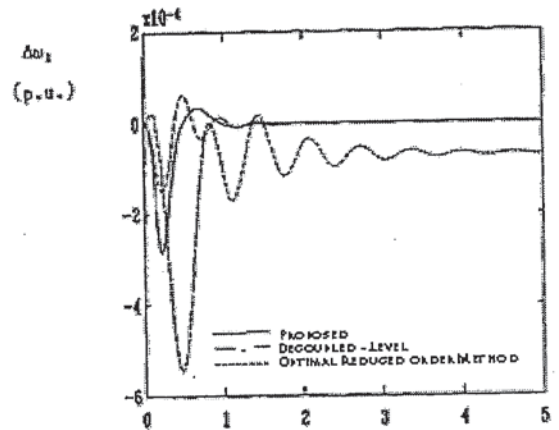


Fig. 4 Transient response following a 5% change in mechanical torque of machine 2.

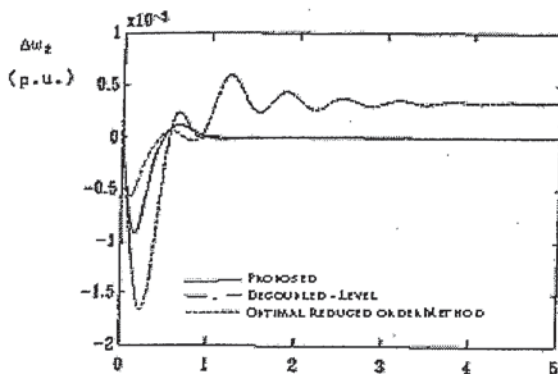
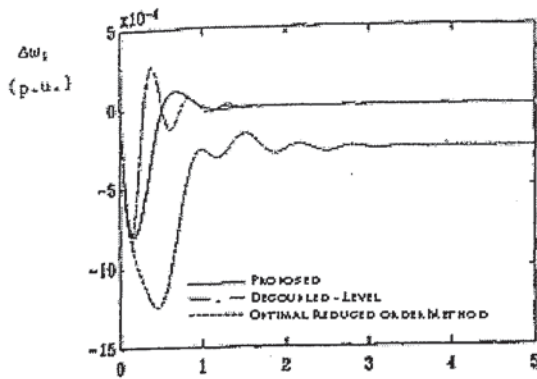


Fig . 5 Transient response following a 5% change in the mechanical torque of both machine at the same time.

From Figure 3-5, it seems like the transient responses of the Decoupled-level method [6] may be better than the proposed method in some cases.

The proposed method is still charming and worthy of suggesting for the following reasons:

1. The decoupled-level method uses all states as control signals , but however the proposed method uses only the output states such as torque angles and speeds.

This takes into account the realities and constraints of the electrical power system and reduces the hardware cost and increases the reliability of the system.

2. From Table II , it is shown that the relative stability of the proposed method is much better than the others. This results from that the optimal reduced order model can retain the worst eigenvalues [4][5].

3. It is simpler to design a PSS with a reduced order model than a whole system model.

5. Conclusion

Optimal output states feedback stabilization of a multi-machine interconnected system is achieved using a two-level control strategy . Local controllers determined at the lower level depend only on local output information pertaining to the particular subsystem. Consequently , a considerable savings in computational time .

Effort at the subsystem level is achieved. The optimal reduced order model is used to retain the physical meaning of the output -states . By using the output feedback only, this approach reduced the implementation cost (hardware) and increases the reliability of the control system. Interaction between the different subsystem is minimized by use of the global controller at a higher level with only output feedback. Thus, a new optimal Decoupled-level control strategy by using only the output states feedback is reached.

The results obtained with the study systems demonstrate that the proposed controller is very effective and has the high relative stability .

6. References

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