Improvement of set-covering Weighted Control Model of S3PR

Daniel Y. Chao[1, a], Yao-Nan Lien[2,b], Jiun-Ting Chen[1], and Yu-Ying Shih[3,c]

[1]Department of Management Information System, [2]Department of Computer Science

National Chengchi University, Taipei, Taiwan, R.O.C.

[3]Department of Information Management, National Taipei College of Business, and

[3] Department of Industrial and Business Management, National Taipei University of Technology,

[a]yuhyaw@gmail.com, [b]lien@cs.nccu.edu.tw, [c]amy.shyy@gmail.com

Keywords: Petri nets, siphons, control, S^3PR.

Abstract The set-covering approach by Piroddi *et al*. may reach the optimal number of states among all approaches for a well-known benchmark using a siphon-based approach without reachability analysis. However, the resulting model is a generalized Petri net incurring extra cost in system verification, validation and implementation. The only improvement is to replace two monitors with weighted arcs by two new monitors without weighted arcs. We develop a theory for explaining the cause of state losses and providing the foundation for the above improvement model.

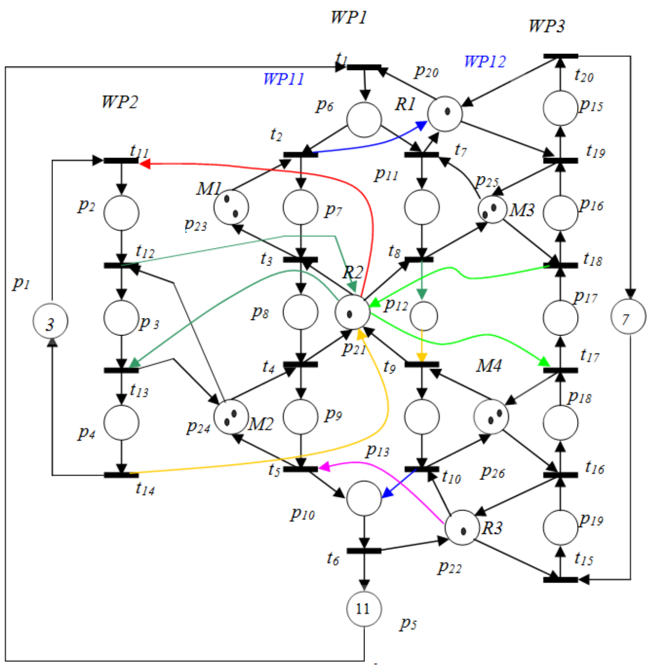
INTRODUCTION

Deadlock prevention has been quite a popular research [1-6]. To avoid deadlocks, monitors and control arcs are added upon emptiable siphons. In the literature there are many deadlock prevention or liveness-enforcing policies proposed for FMS (Flexible Manufacturing Systems), but these methods generally provide suboptimal system behavior resulting in degraded system performance.

Uzam and Zhou [1] applied region analysis (RG) to a well-known S^3PR [3] control policy. They split the RG of the control net into a deadlock-zone (DZ) and a live-zone (LZ). The former may contain deadlock states (markings), partial deadlock states, and states. The latter constitutes remaining good states of the RG representing the optimal system behavior. They further proposed an iterative approach. At each iteration, a first-met bad marking (FBM) is singled out from the reachability graph of a given Petri net model. The objective is to prevent this marking from being reached via a place invariant of the Petri net. This process is carried out until the net model becomes live.

Piroddi *et al.* [2] further increase the 21562 good states by Uzam and Zhou to the optimal 21581 states using the set covering approach for the well-known S^3PR (Fig. 1). A set of siphons is selected by solving a set covering problem during each iteration which explores the relations between uncontrolled siphons and critical markings. By controlling the selective siphons, all the critical markings are forbidden to make all uncontrolled siphons controlled. They do not prove that the policy is maximally permissive in theory. Although they apply Mixed Integer Programming (MIP) to reduce the time to enumerate minimal siphons, the MIP remains to be NP-hard and a number of iterations are required. Furthermore, redundant monitors must be identified in [6] during each iteration, which entails exponential time complexity. Thus, the computational burden remains high and the method is not applicable to large FMS.

Furthermore, quite a few control arcs are weighted rendering the net to be a general Petri net (GPN), which are much harder to analyze. Hence, Piroddi *et al.* transformed weighted arcs into ordinary ones, which sometimes may cause unnecessary deadlocks as mentioned in [4]. It is observed that WC (weighted control arcs) occurs near the end of



iterations. Any further improvement beyond Piroddi *et al.* would reduce the monitors and control arcs, and the simplification of the model with as few weighted control arcs.

Fig. 1. A well-known S^3PR in [9].

This paper explores the cause of state loss and points out the particular siphon responsible for the loss of states. Uzam and Zhou employ a simplified generalized mutual-exclusion constraints (GMECs) equivalently setting the number of tokens in the complementary set $[S]$ of a siphon S fewer than the initial number of tokens in S by one. This excludes some live states where the number of tokens in $[S]$ may equal the initial number of tokens in S. The GMEC by Piroddi *et al.* sets S to be always marked and does not cause states to be lost.

This paper improves the best result by reducing the number of control arcs. The number of weighted control arcs and token count is reduced. This is achieved by replacing two monitors with weighted arcs by two new monitors without weighted arcs. INA (Integrated Net Analyzer) analysis indicates that the controlled model is live and reaches the same states by Piroddi *et al.*

We report an alternative control model [6] of a well-known FMS to reach the same number of good states as that by Uzam *et al.* [1] — the second best in the literature— but with fewer monitors and control arcs by refining some monitors into several, in the later stages of the synthesis, with smaller controller regions. More states can be reached since the controller region is less disturbed by covering only a place in a subregion where only one place is marked at any reachable marking. As a result, the controller region is smaller than the complementary siphon, which, however, may cause the siphon to become unmarked in the initial stages of the synthesis. We propose in this paper to improve Piroddi *et al.*'s result based on the above model and utilize the GMEC by Piroddi *et al.*, which sets S to be always marked and does not cause states to be lost.

IMPROVED MODEL

We report in this paper an alternative control model that uses the same number of monitors but with fewer control arcs (151 compared with 154 in [2]) as well as fewer weighted control arcs (7 compared with 16 in [2]).

A. Past Result

The improved model is based the one reported in [6] and listed in Table I, which is similar to that by Uzam *et al.* except that it reduces several monitors to control one emptiable siphon into a single monitor. Uzam and Zhou select a first-met bad marking, based on region theory, at each iteration from the RA (reachability) of a given control model. Then they add a monitor and control arcs to prevent this bad marking from being reached via a place invariant (PI) based on the GMEC method proposed by Yamalidou *et al.*[6]. This can be achieved by preventing the marking of the subset of the operation places of the FBM from being reached. Continue this process until the net model becomes live.

Note that the above operation place indicates an action to process a part in a production sequence by a resource. Initially there are no tokens in operation places.

Uzam and Zhou consider only the markings of operation places in an FBM, where the number of tokens in the marked operation places Ψ represents the first entry into DZ. Output transitions of these operation places are disabled by some resource places with no tokens. Then, if the marking of Ψ can be prevented from being reached, then the FBM can be forbidden. Therefore, Ψ plus Monitor p_c forms a P-invariant. The maximal sum of tokens within Ψ must be no greater than their current value ℵ within the FBM to not reach the FBM. This can be achieved by setting the initial marking $M_0(p_c)$= ℵ and appropriate control arcs as shown in the following example. For the net in Fig. 1, $2p_{13} + p_{19}$ is an FBM; Ψ={p_{13}; p_{19}} and the number of tokens in Ψ is 3. The GMEC is $M(p_{13})+M(p_{19})$. A monitor p_c is added with $M_0(p_c)$=3-1=2. •p_c=(t_9, t_{15}) and p_c•=(t_{10}, t_{16}). Note that all arc weights are unity. This FBM corresponds to unmarked siphon S_1={p_{10}; p_{18}; p_{22}; p_{26}}. In the next subsection, we will discuss how to apply GMEC and this method to reach more states.

Table I. Refined control model[15] proposed in an earlier paper for the S^3PR in Fig. 1

V_i	•V_i	V_i•	$M_0(V_i)$	S	V_i	•V_i	V_i•	$M_0(V_i)$	S
1	t_{10} t_{16}	t_9 t_{15}	2	basic(S_1)	8	t_8 t_{17}	t_7 t_{16}	3	control SMS
2	t_5 t_{10} t_{13} t_{17}	t_3 t_8 t_{11} t_{15}	5	basic(S_4)	9	t_9 t_{17}	t_7 t_{15}	4	new SMS
3	t_4 t_{13}	t_3 t_{11}	2	basic(S_{10})	10	t_8 t_{10} t_{17}	t_7 t_9 t_{15}	4	new SMS
4	t_{10} t_{17}	t_8 t_{15}	3	compound(S_{15})	11	t_5 t_8 t_{13} t_{17}	t_7 t_3 t_{11} t_{15}	6	new SMS
5	t_9 t_{17}	t_8 t_{16}	2	basic(S_{16})	12	t_5 t_8 t_{10} t_{17} t_{19}	t_1 t_9 t_{15} t_{18}	9	new SMS
6	t_3 t_8 t_{19}	t_1 t_{17}	5	basic(S_{17})	13	t_3 t_5 t_8 t_{10} t_{18}	t_1 t_4 t_9 t_{15}	9	new SMS
7	t_8 t_{18}	t_7 t_{17}	2	basic(S_{18})	14	t_3 t_5 t_9 t_{17} t_{19}	t_1 t_4 t_{15} t_{18}	9	new SMS

C. Application of FBM Method

Consider an FBM for the net in Fig. 1 $p_6+2p_7+p_8+p_9+p_{11}+p_{13}+p_{16}+p_{18}+p_{19}$. Ψ={$p_6$, p_7, p_8, p_9, p_{11}, p_{13}, p_{16}, p_{18}, p_{19}} and the number of tokens in Ψ is 10. A monitor p_c is added with $M_0(p_c)$=10-1=9. The GMEC is G_1= $M(p_6)$+ $M(p_7)$+$M(p_8)$+$M(p_9)$+$M(p_{11})$+$M(p_{13})$+$M(p_{16})$+$M(p_{18})$+ $M(p_{19})$≤9. •p_c=(t_5, t_8, t_{10}, t_{17}, t_{19}) and p_c•=(t_1, t_9, t_{15}, t_{18}). Note that all arc weights are unity. This FBM corresponds to unmarked siphon S^*={p_2, p_3, p_{10}, p_{15}, p_{20}, p_{22}, p_{23}, p_{25}, p_{26}, V_2, V_6} and [S^*]=Ψ. The above GMEC is equivalent to setting $M([S^*])\leq M_0(p_c)$. For this FBM, Piroddi *et al.*, however, employ a different GMEC (based on $M(S^*)\geq 1$ or $-M(S^*)\leq -1$) to guarantee S^* always marked: $G_2=-M(p_2)-M(p_3)-M(p_{10})-M(p_{15})-M(p_{20})-M(p_{22})-M(p_{23})-M(p_{25})-M(p_{26})-M(V_2)-M(V_6)\leq -1$, $\lambda_{S^*} = p_2+p_3+p_{10}+p_{15}+p_{20}+p_{22}+p_{23}+p_{25}+p_{26}+V_2+V_6$, and $\boldsymbol{\eta}^T=\lambda_{S^*}^T\bullet [N^C]=-2t_1+t_3+t_5+t_8-t_9+2t_{10}-2t_{15}+t_{17}-t_{18}+2t_{19}$, which implies •$p_c$=($t_3$, t_5, t_8, $2t_{10}$, t_{17}, $2t_{19}$) and p_c•=($2t_1$, t_9, $2t_{15}$, t_{18}) (Monitor V_{13} in Table I).

Note that [N^C] is a large matrix. Alternatively, one can employ the fact that $\eta(t)$ indicates the number of tokens gained in S^* by firing transition t once. For instance, when t_1 fires once, it grabs a token from each of p_{20} and V_6; hence $\eta(t_1)$=-2. When t_{10} fires once, it grabs a token from p_{22} and deposits a token into each of p_{10}, p_{26} and V_2; hence $\eta(t_1)$=2. Other $\eta(t)$ can be obtained similarly. Note that some control arcs are weighted, while that by Uzam *et al.* are all unit weighted. Replacing the control arcs of Uzam *et al.* for V_{13} by the above one, we get the optimal 21581 states. Thus, the cause of non-maximum-permissiveness of the control model by Uzam *et al.* in [1] is due to the absence of weighted arcs for Monitor V_{13} and the simplified versions of the GMEC G_1 employed by Uzam *et al.* and us. Note that without monitors V_{12} and V_{14}, the net is dead since new emtptiable siphons are generated due to the presence of Monitor V_{13}. One may apply the FBM method to add monitors V_{12} and V_{14} and control arcs, which are the same as that in Table I.

Monitor V_8 is added to siphon $S=\{V_4, V_8, p_{34}, p_{22}, p_{10}, p_{17}\}$ with complementary siphon $[S]=\{p_{11}, p_{12}, p_{13}, p_{18}, p_{19}\}$. The corresponding FBM= $2p_{11}+p_{13}+p_{18}+p_{19}$ and FBM'=$2p_{11}+p_{12}+p_{18}+p_{19}$. $\Psi=\{p_{11},p_{12},p_{18},p_{19}\}$ and $\Psi'=\{p_{11},p_{13},p_{18},p_{19}\}$. The number of tokens in Ψ and Ψ' are both 5. Monitors V_9 and V_{10} are added for Ψ and Ψ' with $M_0(V_9)=M_0(V_{10})=5-1=4$, respectively. The simplified GMEC for Ψ (resp. Ψ') is $G_1=M(p_{11})+M(p_{12})+M(p_{18})+M(p_{19})\leq 4$. ${}^{\bullet}V_9=\{t_9, t_{17}\}$ and $V_9^{\bullet}=\{t_7, t_{15}\}$. Note that all arc weights are unity. This FBM corresponds to unmarked siphon $S=\{V_2,V_4, V_6, V_8, p_{22}, p_{10}, p_{17}\}$. The above GMEC is equivalent to setting $M([S])\leq M_0(p_c)$, where $p_c= V_9$ or V_{10}. For this FBM, Piroddi *et al.*, however, employ a different GMEC (based on $M(S)\geq 1$ or $-M(S) \leq -1$) to guarantee S always marked: $G_2=-M(p_{29})-M(V_4)-M(V_8)-M(p_{10})-M(p_{17})-M(p_{22})\leq -1$, $\lambda_S=p_{10}+p_{17}+p_{22}+p_{29}+V_4+V_8$, and $\boldsymbol{\eta}^T=\lambda_{S^*}{}^T\bullet[N^C]=-2t_7+t_8+t_{10}-2t_{15}+2t_{17}$, which implies $\bullet p_c=(t_8, t_{10}, 2t_{17})$ and $p_c\bullet=(2t_7, t_9, 2t_{15})$.Again, we employ the fact that $\eta(t)$ indicates the number of tokens gained in S by firing transition t once to avoid constructing the large incidence matrix $[N^C]$.

Our better results arise from the fact that we replace monitors V_{12} and V_{14} with weighted arcs by two monitors with same indexes but without weighted arcs. We observe that the arcs from these two monitors to t_{11} and those from t_{13} to these two monitors can be eliminated without affecting the liveness of the control model. Another advantage of our model is that the above monitors carry high token accounts of 27 and 22 in [4] compared with our 9 and 9, respectively. This may be an advantage since in general higher token accounts imply larger space of reachable markings.

CONCLUSION

We have proposed a slightly better control policy to reach the same optimal number of 21581 states, where weighted arcs are associated with only one monitor rather than 4 monitors by Piroddi *et al.*, but one more monitor is required and also with 3 fewer control arcs and 9 fewer weighted arcs, yielding a simpler control model. Our optimal results have been achieved by judiciously choosing the sets of siphons to perform simplified generalized GMEC and the rest to perform the regular GMEC by Piroddi *et al.* It remains a future research to develop a systematic approach for the above selection.

REFERENCES

[1] M. Uzam, M.C. Zhou: An iterative synthesis approach to Petri net based deadlock prevention policy for flexible manufacturing systems. IEEE Transactions on Systems, Man and Cybernetics - Part A: Systems and Humans, 37 (2007), p. 362-371

[2] L. Piroddi, R. Cordone, and I. Fumagalli: Selective Siphon Control for Deadlock Prevention in Petri Nets. IEEE Transaction on Systems Man and Cybernetic Part A Systems and Humans, 38(6) (2008), p.1337-1348

[3] J. Ezpeleta, J.M. Colom, and J. Martinez: A Petri net based deadlock prevention policy for flexible manufacturing systems. IEEE Trans. Robot. Automat., 11(2) (1995), p. 173-184

[4] D. Y. Chao: A Simple Modification of Deadlock Prevention Policy of S^3PR Based on Elementary Siphons. To appear in Trans. Mea. and Instru., (2009)

[5] D. Y. Chao, G.J. Liu, and J.J. Chang: A Simple Suboptimal Siphon-Based Control Model of A Well-Known S^3PR, Asian Control of Journal., DOI: 10.1002/asjc.292, 2010, 07. (2009)

[6] K. Yamalidou, J. Moody, M. Lemmon, and P. Antsaklis: Feedback control of Petri nets based on place invariants, *Automatic*, 32(1) (1996), p. 15-28