## 政大在職碩班「專題研討」

## Deniable Encryption

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## Who am I？

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## Outline

- Introduction to Deniable Encryption.
- Shared-Key Deniable Encryption.
- Public-Key Deniable Encryption.
- Bi-Deniable Public-Key Deniable Encryption.
- Block-wise Deniable Encryption.
- Reference.


## Don't Worry.

This is not a mathematic course.
Everything will be described in plain Chinese.

# Introduction to Deniable Encryption 

## Deniable Encryption



## Deniable Encryption



Deniable Encryption!!

## This is a Stupid Scenario!!

Actually, this is a real story.

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## Email Service Provider

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Lavabit


On January 20, 2017, Lavabit owner Ladar Levison relaunched the service.

## Deniable Encryption

- An encryption scheme is deniable if the entities can generate plausible keys or random coins that will satisfy the authority.
- Usage: Protect people from subpoenas or legal coercion.
- Ex: E-Voter, Journalist, Whistle-blowers.
- Theoretical Properties:
- Non-committing.
- Against selective-opening attacks.
- Incoercible multi-party computation.


## Selective Opening Attack

- Given a public key encryption scheme:
- $c=(c[1], c[2], c[3], \ldots, c[n])$.
- $c[i]=E(p k, m[i], r[i]), 1 \leq i \leq n$.
- All coins $r[i]$ are random and independent.
- The adversary is allowed to corrupt some subset $I$ :
- $r[i], i \in I$.
- $m[i], i \in I$.
- The security requirement is that the privacy of the unopened messages is preserved.


## Some Definitions

## Definition

## Computational Indistinguishable:

Let $A=\left\{A_{n}\right\}_{n \in N}$ and $B=\left\{B_{n}\right\}_{n \in N}$ be two probability distributions and $\delta: N \rightarrow[0,1]$. $A$ and $B$ are $\delta(n)$-close if for every polytime distinguisher $D$ and for all large enough $n$,

$$
\left|\operatorname{Prob}\left(D\left(A_{n}\right)=1\right)-\operatorname{Prob}\left(D\left(B_{n}\right)=1\right)\right|<\delta(n)
$$

If $\delta(n)$ is negligible, $A$ and $B$ are computational indistinguishable and write $A \approx^{c} B$.

## Definition

## Correctness:

The probability that $R$ 's output is different than $S^{\prime} s$ input is negligible (as a function of $n$ ).

## Some Definitions

## Definition

## Plan-Ahead:

A somewhat weaker deniability property allows the encryption algorithm to have the fake messages as part of its input.

## Definition

## Sender Deniable:

1. Correctness.
2. Security: $E\left[m_{1}\right] \approx^{c} E\left[m_{2}\right]$.
3. Deniability:

- $c=E\left[m_{1}, r_{S}\right]$.
- A faking algorithm $\phi$ that $r_{s}^{\prime}=\phi\left(m_{1}, r_{s}, c, m_{2}\right)$.
- $\left(m_{2}, r_{s}^{\prime}, c\right) \approx^{c}\left(m_{2}, r_{s}^{\prime \prime}, E\left[m_{2}, r_{s}^{\prime \prime}\right]\right)$.


## Some Definitions

## Definition

## Receiver Deniable:

1. Correctness.
2. Security: $E\left[m_{1}\right] \approx^{c} E\left[m_{2}\right]$.
3. Deniability:

- $c=E\left[m_{1}, r_{R}\right]$.
- A faking algorithm $\phi$ that $r_{R}^{\prime}=\phi\left(m_{1}, r_{R}, c, m_{2}\right)$.
- $\left(m_{2}, r_{R}^{\prime}, c\right) \approx^{c}\left(m_{2}, r_{R}^{\prime \prime}, E\left[m_{2}, r_{R}^{\prime \prime}\right]\right)$.


## Shared-Key Deniable Encryption

## Shared-Key Deniable Encryption

- The most trivial solution is: One Time Pad, Vernam Cipher.
- $c \leftarrow m \oplus k$.
- $k^{\prime} \leftarrow c \oplus m^{\prime}$.
- $m^{\prime}$ can be chosen as late as at time of coercion.
- This scheme is not practical for most cases.


## Shared-Key Deniable Encryption based on Pseudorandom Generators

- The message will be encrypted:
- $m_{1}=m_{1}^{(1)}, m_{1}^{(2)}, m_{1}^{(3)}, \ldots$.
- Each block $m_{1}^{(j)}$ is $n$-bit.
- The fake messages:
- $m_{2}=m_{2}^{(1)}, m_{2}^{(2)}, m_{2}^{(3)}, \ldots$.
- $m_{l}=m_{l}^{(1)}, m_{l}^{(2)}, m_{l}^{(3)}, \ldots$
- The shared key:
- $k_{1}$ : n-bit random key.
- $k_{2}, \ldots, k_{l}$ : l-1 independent $n$-bit fake keys.
- A pseudorandom number generator $G$ :
- Expand n-bit input to $3 n$-bit output.
- Using $G$ iteratively: $G\left(k_{i}^{(j-1)}\right)=k_{i}^{(j)}\left|a_{i}^{(j)}\right| b_{i}^{(j)}$.


## Shared-Key Deniable Encryption based on Pseudorandom Gen-

 erators- Encryption:
- $c=c^{(1)}, c^{(2)}, c^{(3)}, \ldots$
- The sender finds the polynomial $Q^{(j)}$ of degree $/-1$ such that $Q^{(j)}\left(a_{i}^{(j)}\right)=m_{i}^{(j)}+b_{i}^{(j)}, i=1 \ldots l$.
- $c^{(j)}=\left\langle j, Q^{(j)}\right\rangle$.
- Decryption:
- $m_{1}^{(j)}=Q^{(j)}\left(a_{1}^{(j)}\right)-b_{1}^{(j)}$.
- Deniability:
- Just select one of fake keys when coercion.


## Public-Key Deniable Encryption

## Translucent Set

- This scheme is based on the trapdoor SPARSE sets.

1. A small set $S \subset\{0,1\}^{t},|S| \leq 2^{t-k}$ for some $k$.
2. It is easy to generate random element $x \in S$.
3. Without the trapdoor $d$, it is infeasible to decide whether $x \in\{0,1\}^{t}$ was chosen from $S$ or uniformly from $\{0,1\}^{t}$.


## How to Construct Sparse Sets

- A trapdoor permutation $f:\{0,1\}^{s} \rightarrow\{0,1\}^{s}$.
- A hard-core bit function $B:\{0,1\}^{s} \rightarrow\{0,1\}$.
- Construction I:
- $t=s k$.
- Represent $x \in\{0,1\}^{t}$ as a vector $x=x_{1} x_{2} \ldots x_{k}$, where $x_{i} \in\{0,1\}^{s}$.
- $S=\left\{x \in\{0,1\}^{s k} \mid \forall i=1 \ldots k, B\left(f^{-1}\left(x_{i}\right)\right)=0\right\}$.
- $|S|=2^{(s-1) k}=2^{t-k}$.


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- $|S|=2^{(s-1) k}=2^{t-k}$.
- Construction II:
- $t=s+k$.
- Represent $x \in\{0,1\}^{t}$ as a vector $x=x_{0}, b_{1} b_{2} \ldots b_{k}$, where $x_{0} \in\{0,1\}^{s}$ and $b_{i} \in\{0,1\}$.
- $S=\left\{x \in\{0,1\}^{s+k} \mid \forall i=1 \ldots k, B\left(f^{-i}\left(x_{0}\right)\right)=b_{i}\right\}$.
- $|S|=2^{s}=2^{t-k}$.


## Public-Key Sender-Deniable Encryption Scheme 01

- The Basic Scheme:
- Bitwise encryption.
- Public key: $S \subset\{0,1\}^{\text {t }}$; Private key: the trapdoor $d$.
- Encryption:
- To encrypt 1 , send a random element from $S$.
- To encrypt 0 , send a random element from $\{0,1\}^{t}$.
- Decryption: Check if the cipher $c$ is in $S$.
- Dinability: If the encrypted bit is 1 , claim that the cipher is chosen from $\{0,1\}^{t}$ instead from $S$.
- Only half deniablity.
- The probability of decryption error is $\frac{2^{t-k}}{2^{t}}=2^{-k}$.


## Public-Key Sender-Deniable Encryption Scheme 02

- The Parity Scheme:
- Public key: $S \subset\{0,1\}^{t}, R=\{0,1\}^{\text {t }}$; Private key: the trapdoor d.
- Use $V \in\{S, R\}^{n}$ to denote a length $n$ vector.
- Encryption:
- To encrypt 1 , send a $V \in_{R}\{S, R\}^{n}$ where $V$ randomly contains odd $S$-elements.
- To encrypt 0 , send a $V \in_{R}\{S, R\}^{n}$ where $V$ randomly contains even $S$-elements.
- Decryption: Reveal the number of elements in $V$ that belongs to $S$.
- Deniability: The sender can claim $V$ has $i-1 S$-elements rather than $i$.
- The probability of decryption error is at most $n 2^{-k}$.


## Receiver-Deniability and Bi-Deniability

- Receiver-Deniability from Sender-Deniability:
- If there is a Sender-Deniable scheme, the receiver first sends a deniable message $r$ to the sender.
- The sender sends $m \oplus r$ to the receiver.
- Bi-Deniability:
- Sender-and-Receiver-Deniability.
- $\oplus_{i} b_{i}=b$.
- As long as one intermediary node is uncoerced, the sender and the receiver can deny their messages.



## What is the Problem?

- We do not like bitwise encryption.
- We do not like interactive encryption.
- We do not like third-party.
- We do not like decryption error.


## Bi-Deniable Public-Key Deniable Encryption

## Multi-Distributional Bi-Deniable Scheme

- A. O'Neil, C. Peikert and B. Waters proposed Multi-Distributional Bi-Deniable Scheme based on Simulatable Public-Key Encryption.


## Definition

## Multi-Distributional:

- Multi-Distributional means the parties run alternative key-generation and encryption algorithms for equivocable communication, but claim under coercion to have run the prescribed algorithms.
- Multi-Distributional means the scheme contains normal and deniable encryption at the same time.


## A Philosophical Question

- Why would anyone ever choose to send a message according to the non-deniable encryption algorithm?
- It is impossible to eliminate this option because the coercer would know that the sender is lying.
- The purpose of deniability is not at all to convince the coercer, but to preempt coercion in the first place.


## Bi-Deniable Scheme

| Sender-Deniable | Receiver-Deniable |
| :--- | :--- |
| $p k \leftarrow \operatorname{Gen}\left(1^{n}, r_{R}\right)$ | $(p k, f k) \leftarrow \operatorname{DenGen}\left(1^{n}\right)$ |
| $c \leftarrow \operatorname{DenEnc}\left(p k, m, r_{S}\right)$ | $c \leftarrow \operatorname{Enc}\left(p k, m, r_{S}\right)$ |
|  | $r_{R}^{*} \leftarrow \operatorname{RecFake}\left(p k, f k, c, m^{\prime}\right)$ |
| $r_{S}^{*} \leftarrow \operatorname{SendFake}\left(p k, r_{S}, m, m^{\prime}\right)$ | $\left(p k, c, r_{R}^{*}\right)$ |
| Return $\left(p k, c, r_{S}^{*}\right)$ |  |

Bi-Deniable
$(p k, f k) \leftarrow \operatorname{DenGen}\left(1^{n}\right)$
$c \leftarrow \operatorname{DenEnc}\left(p k, m, r_{s}\right)$
$r_{R}^{*} \leftarrow \operatorname{RecFake}(p k, f k, c, b)$
$r_{S}^{*} \leftarrow \operatorname{SendFake}\left(p k, r_{S}, m, m^{\prime}\right)$
Return ( $p k, c, r_{S}^{*}, r_{R}^{*}$ )

## Simulatable Public-Key System

## Definition

Given a public-key system ( $K, E, D, M$ ), where

- K: key generation algorithm; E: encryption algorithm;
- D: decryption algorithm; $M$ : message space generator.
$(K, E, D, M)$ is a simulatable public key system if $\left(\tilde{K}, \tilde{K}^{-1}, C, C^{-1}\right)$ exists:
- Oblivious public key generation:

$$
\begin{gathered}
r \leftarrow R,(P, S) \leftarrow K(r), r^{\prime} \leftarrow \tilde{K}^{-1}(P) \\
r^{\prime \prime} \leftarrow R,\left(P^{\prime \prime}, S^{\prime \prime}\right) \leftarrow \tilde{K}\left(r^{\prime \prime}\right)
\end{gathered}
$$

$\left(r^{\prime}, P\right)$ and $\left(r^{\prime \prime}, P^{\prime \prime}\right)$ are computationally indistinguishable.

## Simulatable Public-Key System

## Definition

- Oblivious ciphertext generation:

$$
\begin{aligned}
& \qquad \qquad(P, S) \leftarrow K, r_{1} \leftarrow R, C_{1} \leftarrow C\left(P, r_{1}\right) \\
& r_{2} \leftarrow R, C_{2} \leftarrow E_{P}\left(M, r_{2}\right), r_{2}^{\prime} \leftarrow C^{-1}\left(C_{2}, P\right) . \\
& \left(P, r_{1}, C_{1}\right) \text { and }\left(P, r_{2}^{\prime}, C_{2}\right) \text { are computationally } \\
& \text { indistinguishable. }
\end{aligned}
$$

## Simulatable Public-Key System

## Definition

- Semantic security:

$$
\begin{aligned}
& \qquad \begin{array}{l}
r \leftarrow R,(P, S) \leftarrow K(r) \\
r_{0} \leftarrow R, C_{0} \leftarrow E_{P}\left(M_{0}, r_{0}\right) \\
r_{1} \leftarrow R, C_{1} \leftarrow E_{P}\left(M_{1}, r_{1}\right) \\
\left(P, M_{0}, M_{1}, C_{0}\right) \text { and }\left(P, M_{0}, M_{1}, C_{1}\right) \text { are computationally } \\
\text { indistinguishable. }
\end{array} \text { }
\end{aligned}
$$

## ElGamal is Simulatable under DDH Assumption

- ElGamal Encryption:
- Public key: $h=g^{x}, p, g$.
- Private key: x.
- Encryption: $\left(g^{y}, m h^{y}\right)$.
- Oblivious:
- $\tilde{K}=h$.
- $\tilde{K}^{-1}(p, g, h)=(p, g, h)$.
- $C=\left(y_{1}, y_{2}\right)$, where $y_{1} \leftarrow R, y_{2} \leftarrow R$.


## Bideniable Encryption from Simulatable System (1/3)

| BI-DEN.Gen $\left(1^{n}\right)$ | BI-DEN.Enc $(p k, b)$ |
| :--- | :--- |
| $R \leftarrow P_{n}([5 n])$ | $S \leftarrow P_{n}([5 n])$ |
| For $i=1$ to $5 n$ do: | For $i=1$ to $5 n$ do: |
| If $i \in R$ then | If $i \in S$ then |
| $\quad p k_{i} \leftarrow \operatorname{Gen}\left(1^{n}, r_{R, i}\right)$ | $c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, b, r_{s, i}\right)$ |
| Else | Else |
| $p k_{i} \leftarrow \operatorname{OGen}\left(1^{n}, r_{R, i}\right)$ | $c_{i} \leftarrow \operatorname{OEnc}\left(p k_{i}, r_{S, i}\right)$ |
| $p k \leftarrow p k_{1}\left\\|p k_{2}\right\\| \ldots \\| p k_{5 n}$ | $c \leftarrow c_{1}\left\\|c_{2}\right\\| \ldots \\| c_{5 n}$ |
| Return $p k$ | Return $c$ |

$$
\begin{aligned}
& \text { BI-DEN. } \operatorname{Dec}\left(\left(R, r_{R}\right), c\right) \\
& \hline \text { For } i \in R \text { do: } \\
& \quad d_{i} \leftarrow \operatorname{Dec}\left(r_{R, i}, c_{i}\right) \\
& \text { If most } d_{i}^{\prime} \text { 's are } 1 \text { then Return } 1 \\
& \text { Else Return } 0
\end{aligned}
$$

## Voting

Encryption.


## Voting

Encryption.


## Voting

## Decryption.



## Proof of Correctness

- BI-DEN.Enc should be correct.
- The tail of the hypergeometric distribution:

$$
\begin{aligned}
& \operatorname{Pr}\left[X \leq E[X]-t y=y\left(\frac{M}{N}-t\right)\right] \leq e^{-2 t^{2} y} \\
& \operatorname{Pr}\left[X \leq E[X]+t y=y\left(\frac{M}{N}+t\right)\right] \leq e^{-2 t^{2} y}
\end{aligned}
$$

- BI-DEN.Enc:
- Let $/$ be $|S \cap R|$ and $D$ be $R \backslash S$ and $d_{i}=b$.
- Decryption error: $D+I<\frac{n}{2}$.
- If $\frac{n}{10}<I \leq \frac{n}{2}$,

$$
\operatorname{Pr}\left[D \leq \frac{n-1}{2}-\frac{l}{2}\right] \leq \operatorname{Pr}\left[D \leq\left(1-\frac{1}{9}\right) E[D]\right] \leq e^{-\frac{n-1}{324}} \leq e^{-\frac{n}{648}}
$$

## Bideniable Encryption from Simulatable System (2/3)

| BI-DEN.DenGen $\left(1^{n}\right)$ | BI-DEN.DenEnc $(p k, b)$ |
| :--- | :--- |
| $R \leftarrow P_{n}([5 n])$ | $S_{0} \leftarrow P_{n}([5 n])$ |
| For $i=1$ to $5 n$ do: | $S_{1} \leftarrow P_{n}\left([5 n] \backslash S_{0}\right)$ |
| $p k_{i} \leftarrow \operatorname{Gen}\left(1^{n}, r_{R, i}\right)$ | $Y \leftarrow P_{n}\left([5 n] \backslash\left(S_{0} \cup S_{1}\right)\right)$ |
| $p k \leftarrow p k_{1}\left\\|p k_{2}\right\\| \ldots \\| p k_{5 n}$ | For $i=1$ to $5 n$ do: |
| $r \leftarrow r_{R, 1}\left\\|r_{R, 2}\right\\| \ldots \\| r_{R, 5 n}$ | If $i \in S_{0}$ then $c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, 0, r_{S, i}\right)$ |
|  | If $i \in S_{1}$ then $c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, 1, r_{S, i}\right)$ |
|  | If $i \in Y$ then $c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, b, r_{S, i}\right)$ |
| Return $(p k,(R, r))$ | Else $c_{i} \leftarrow \operatorname{OEnc}\left(p k_{i}, r_{S, i}\right)$ |
|  | $c \leftarrow c_{1}\left\\|c_{2}\right\\| \ldots \\| c_{5 n}$ |
|  | Return $c$ |

## Bideniable Encryption from Simulatable System (3/3)

BI-DEN.FakeCoins( $p k, f k, r_{s}, b, b^{\prime}$ )
$c \leftarrow \operatorname{BI}-D E N . E n c\left(p k, b, r_{S}\right)$
$z \leftarrow \operatorname{HGD}(5 n, n, n)$
$Z \leftarrow P_{z}\left(S_{b^{\prime}}\right)$
$Z^{\prime} \leftarrow P_{n-z}\left([5 n] \backslash\left(S_{0} \cup S_{1} \cup Y\right)\right.$
$R^{*} \leftarrow Z \cup Z^{\prime}$
$S^{*} \leftarrow S_{b^{\prime}}$
For $i=1$ to $5 n$ do:
If $i \in S^{*}$, then $r_{S, i}^{*} \leftarrow r_{S, i}$
Else $r_{S, i}^{*} \leftarrow l_{\text {OEnc }}\left(p k_{i}, c_{i}\right)$
If $i \in R^{*}$, then $r_{R, i}^{*} \leftarrow r_{R, i}$
Else $r_{R, i}^{*} \leftarrow l_{\text {OGen }}\left(p k_{i}\right)$
$r_{S}^{*} \leftarrow r_{S, 1}^{*}\left\|r_{S, 2}^{*}\right\| \ldots \| r_{S, 5 n}^{*}$
$r_{R}^{*} \leftarrow r_{R, 1}^{*}\left\|r_{R, 2}^{*}\right\| \ldots \| r_{R, 5 n}^{*}$
Return $\left(r_{\varsigma}^{*}, r_{R}^{*}\right)$

- Hypergeometric

Distribution:

$$
\begin{aligned}
& P_{\text {HGD }}(x, N, M, y)= \\
& \frac{C_{x}^{N} C_{y-M}^{N-x}}{C_{y}^{N}} .
\end{aligned}
$$

- $\operatorname{HGD}(N, M, y)$ is the expectation.


## Cheating

Claim.


## Cheating

In fact.


## Proof of Deniability (1/3)

| Experiment $G_{0}$ |
| :--- |
| $S_{b} \leftarrow P_{n}([5 n])$ |
| $R \leftarrow P_{n}([5 n])$ |
| $S_{1-b} \leftarrow P_{n}\left([5 n] \backslash\left(S_{b} \cup R\right)\right)$ |
| $Y \leftarrow P_{n}\left([5 n] \backslash\left(S_{b} \cup S_{1-b} \cup R\right)\right)$ |

For $i=1$ to $5 n$ do:
If $i \in R, p k_{i} \leftarrow \operatorname{Gen}\left(1^{n}, r_{R, i}\right)$
Else $p k_{i} \leftarrow \operatorname{OGen}\left(1^{n}, r_{R, i}\right)$
If $i \in S_{b}, c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, b, r_{S, i}\right)$
Else $c_{i} \leftarrow \operatorname{OEnc}\left(p k_{i}, r_{S, i}\right)$
Return (nk $\left.\left(R r_{0}\right)\left(S, r_{0}\right)\right)$

$$
\begin{aligned}
& \text { Experiment } G_{1} \\
& S_{b} \leftarrow P_{n}([5 n]) \\
& z \leftarrow H G D(5 n, n, n) \\
& Z \leftarrow P_{z}\left(S_{b}\right) \\
& Z^{\prime} \leftarrow P_{n-z}\left([5 n] \backslash S_{b}\right) \\
& R \leftarrow Z \cup Z^{\prime} \\
& S_{1-b} \leftarrow P_{n}\left([5 n] \backslash\left(S_{b} \cup R\right)\right) \\
& Y \leftarrow P_{n}\left([5 n] \backslash\left(S_{b} \cup S_{1-b} \cup R\right)\right)
\end{aligned}
$$

For $i=1$ to $5 n$ do:
If $i \in R, p k_{i} \leftarrow \operatorname{Gen}\left(1^{n}, r_{R, i}\right)$
Else $p k_{i} \leftarrow \operatorname{OGen}\left(1^{n}, r_{R, i}\right)$
If $i \in S_{b}, c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, b, r_{S, i}\right)$
Else $c_{i} \leftarrow \operatorname{OEnc}\left(p k_{i}, r_{S, i}\right)$
Return (nk $\left.\left(R r_{0}\right)\left(S, r_{0}\right)\right)$

## Proof of Deniability (2/3)

| Experiment $G_{1}$ |
| :--- |
| $S_{b} \leftarrow P_{n}([5 n])$ |
| $z \leftarrow H G D(5 n, n, n)$ |
| $Z \leftarrow P_{z}\left(S_{b}\right)$ |
| $Z^{\prime} \leftarrow P_{n-z}\left([5 n] \backslash S_{b}\right)$ |
| $R \leftarrow Z \cup Z^{\prime}$ |
| $S_{1-b} \leftarrow P_{n}\left([5 n] \backslash\left(S_{b} \cup R\right)\right)$ |
| $Y \leftarrow P_{n}\left([5 n] \backslash\left(S_{b} \cup S_{1-b} \cup R\right)\right)$ |
| For $i=1$ to $5 n$ do: |
| If $i \in R, p k_{i} \leftarrow \operatorname{Gen}\left(1^{n}, r_{R, i}\right)$ |
| Else $p k_{i} \leftarrow \operatorname{OGen}\left(1^{n}, r_{R, i}\right)$ |
| If $i \in S_{b}, c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, b, r_{S, i}\right)$ |
| Else $c_{i} \leftarrow \operatorname{OEnc}\left(p k_{i}, r_{S, i}\right)$ |

Experiment $G_{2}$

$$
\begin{aligned}
& S_{b} \leftarrow P_{n}([5 n]) \\
& S_{1-b} \leftarrow P_{n}\left([5 n] \backslash S_{b}\right) \\
& Y \leftarrow P_{n}\left([5 n] \backslash\left(S_{b} \cup S_{1-b}\right)\right) \\
& z \leftarrow H G D(5 n, n, n) \\
& Z \leftarrow P_{z}\left(S_{b}\right) \\
& Z^{\prime} \leftarrow P_{n-z}\left([5 n] \backslash\left(S_{b} \cup S_{1-b} \cup Y\right)\right. \\
& R \leftarrow Z \cup Z^{\prime}
\end{aligned}
$$

For $i=1$ to $5 n$ do:
If $i \in R, p k_{i} \leftarrow \operatorname{Gen}\left(1^{n}, r_{R, i}\right)$
Else $p k_{i} \leftarrow \operatorname{OGen}\left(1^{n}, r_{R, i}\right)$
If $i \in S_{b}, c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, b, r_{s, i}\right)$
Else $c_{i} \leftarrow \operatorname{OEnc}\left(p k_{i}, r_{s, i}\right)$

Experiment $G_{2}$
For $i=1$ to $5 n$ do:
If $i \in R, p k_{i} \leftarrow \operatorname{Gen}\left(1^{n}, r_{R, i}\right)$
Else $p k_{i} \leftarrow \operatorname{OGen}\left(1^{n}, r_{R, i}\right)$
If $i \in S_{b}, c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, b, r_{S, i}\right)$
Else $c_{i} \leftarrow \operatorname{OEnc}\left(p k_{i}, r_{S, i}\right)$

## Experiment $G_{3}$

For $i=1$ to $5 n$ do:

$$
\begin{aligned}
& p k_{i} \leftarrow \operatorname{Gen}\left(1^{n}, r_{R, i}\right) \\
& \text { If } i \in R, r_{R, i}^{*} \leftarrow r_{R, i}
\end{aligned}
$$

Else $r_{R, i}^{*} \leftarrow \mathrm{I}_{\mathrm{OGen}}\left(p k_{i}\right)$
If $i \in S_{b}$

$$
c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, b, r_{S, i}\right), r_{S, i}^{*} \leftarrow r_{S, i}
$$

Else if $i \in S_{1-b}$

$$
\begin{aligned}
& c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, 1-b, r_{S, i}\right) \\
& r_{S, i}^{*} \leftarrow \operatorname{IOEnc}\left(p k_{i}, c_{i}\right)
\end{aligned}
$$

Else if $i \in Y$

$$
\begin{aligned}
& c_{i} \leftarrow \operatorname{Enc}\left(p k_{i}, b^{\prime}, r_{S, i}\right) \\
& r_{S, i}^{*} \leftarrow \operatorname{IOEnc}\left(p k_{i}, c_{i}\right)
\end{aligned}
$$

Else
$c_{i} \leftarrow$ OEnc $\left(p k_{i}, r_{s}\right), r_{c}^{*} \leftarrow r_{\mathrm{s}}$

## Question

How does the receiver know $S_{0}, S_{1}, Y$ ?

## Block-wise Deniable Encryption

## Problems

- The proposed schemes are bitwise.
- Cost too much.
- Consistency issue.


## Plan-Ahead Bi-Deniable Encryption (1/3)

| $\operatorname{Gen}\left(1^{n}\right):$ | $\operatorname{Enc}(p k, m):$ |
| :--- | :--- |
|  |  |
| $(p k, s k) \leftarrow \operatorname{Gen}^{\prime}\left(1^{n}\right)$ | $K_{0} \leftarrow\{0,1\}^{n}, b \leftarrow\{0,1\}$ |
| Return $(p k, s k)$ | $c_{\text {asym }} \leftarrow \operatorname{Enc}^{\prime}\left(p k, K_{0}\left\\|0^{n}\right\\| b\right)$ |
|  | $c_{0} \leftarrow E\left(K_{0}, m\right)$ |
|  | $c_{1} \leftarrow\{0,1\}\left\|c_{b}\right\|$ |
|  | Return $c_{\text {asym }}\left\\|c_{b}\right\\| c_{1-b}$ |

## Plan-Ahead Bi-Deniable Encryption (2/3)

| DenGen $\left(1^{n}\right):$ | PADenEnc $\left(p k, m_{0}, m_{1}\right):$ |
| :--- | :--- |
|  |  |
| $(p k, s k, f k) \leftarrow \operatorname{Gen}^{\prime}\left(1^{n}\right)$ | $K_{0}, K_{1} \leftarrow\{0,1\}^{n}, b \leftarrow\{0,1\}$ |
| Return $(p k, s k, f k)$ | $c_{\text {asym }} \leftarrow \operatorname{DenEnc} c^{\prime}\left(p k, K_{0}\left\\|K_{1}\right\\| b\right)$ |
|  | $c_{0} \leftarrow E\left(K_{0}, m_{0}\right)$ |
|  | $c_{1} \leftarrow E\left(K_{1}, m_{1}\right)$ |
|  | $\operatorname{Return} c_{\text {asym }}\left\\|c_{b}\right\\| c_{1-b}$ |

## Plan-Ahead Bi-Deniable Encryption (3/3)

| PARecFake $\left(f k, c, K_{0}\left\\|K_{1}\right\\| b, b^{\prime}\right):$ | PASendFake $\left(p k, c, r_{S}, b^{\prime}\right):$ |
| :--- | :--- |
|  |  |
| $c \leftarrow c_{\text {asym }}\left\\|c_{0}\right\\| c_{1}$ | $c \leftarrow c_{\text {asym }}\left\\|c_{0}\right\\| c_{1}$ |
| $x \leftarrow K_{0}\left\\|K_{1}\right\\| b$ | $K_{0}\left\\|K_{1}\right\\| b \\| r \leftarrow r_{S}$ |
| $y \leftarrow K_{b^{\prime}}\left\\|0^{n}\right\\| b^{\prime}$ | $x \leftarrow K_{0}\left\\|K_{1}\right\\| b$ |
| $r_{R}^{*} \leftarrow \operatorname{RecFake}^{\prime}\left(f k, c_{a s y m}, x, y\right)$ | $y \leftarrow K_{b^{\prime}}\left\\|0^{n}\right\\| b^{\prime}$ |
| $\operatorname{Return} r_{R}^{*}$ | $r_{S}^{*} \leftarrow \operatorname{SendFake}^{\prime}\left(p k, c_{a s y m}, r, x, y\right)$ |

## Chameleon Hash



Chameleon Hash is a trapdoor one-way function with three requirements:

1. Semantic Security.
2. Collision Resistance.
3. Collision Forgery with the trapdoor.

Most trapdoor pseudo random permutation functions can be used as chameleon hash functions.

## Ciphertext Pattern

## C1 <br> C2 <br> Ciphertext Arguments



- Normal Ciphertext: $V=C H\left(t_{b}, M\right)$.
- Deniable Ciphertext: $V=C H\left(t_{b}, M\right)=C H\left(t_{1-b}, M^{*}\right)$.

Note: $b$ can be used as a sender proof.

## Reference

## Reference

1. R. Canetti, C. Dwork, M. Naor and R. Ostrovsky. Deniable Encryption. Crypto 1997.
2. A. O'Neil, C. Peikert and B. Waters. Bi-Deniable Public-Key Encryption. Crypto 2011.
3. P. Chi and C. Lei. Audit-Free Cloud Storage via Deniable Attribute-Based Encryption. IEEE TCC 2018.
$Q$ and $A$

