**等候理論/排隊理論 期末考**

6/21/2019

[題目共3面，7題，滿分70分] 時間： 9:10~11:00am

[你可以帶一張A4紙的小抄進來考試，除此之外的東西不准翻閱]

[10%] 1. Consider a system with two identical machines that are not maintained by any repairman. Each machine functions for an exponential time with mean 1/αbefore breaking down. If only one is operational, the system still can work. However, if the two machines are both down, the system is considered to be crashed. Let state *i* indicates that *i* machines are operational. Draw the Markov Chain, and find the Laplace transform for the life time of the system. (We assume that system starts with 2 operational machines)

[10%] 2. (Problem 4.9) Consider an M/M/1 system with the following variation: Whenever the server becomes free, he accept *two* customers (if at least two are available) from the queue into service simultaneously. Of the two customers, only one receives service; when the service for this one is completed, both customers depart (and so the other customer got a “free ride”). If only one customer is available in the queue when the server becomes free, then that customer is accepted alone and is serviced; if a new customer happens to arrive when this single customer is being served, then the new customer joins the old one in service and this new customer receives a “free ride”.

In all cases, the service time is exponentially distributed with mean 1/μ sec and the average (Poisson) arrival rate is λ customers per second.

1. Draw the appropriate state diagram.
2. Write down the appropriate difference equations for *pk* = equilibrium probability of finding *k* customers in the system.
3. Solve for *P(z)* in terms of *p0* and *p1*.

[10%] 3. Consider a taxi station where taxis and customers arrive in accordance with Poisson processes with respectively rates of 1 and 2 per minute. A taxi will wait no matter how many other taxis are present. However, if an arriving customer does not find a taxi waiting, **and there is already one customer waiting ahead of him,** he leaves. If no other customer is waiting, he waits. (i.e. One waiting rooms). Assume a taxi only takes 1 customer. Find

1. the average number of taxis waiting
2. the proportion of arriving customers that get taxis.
3. average waiting time of arriving customers that get taxis.

[10%] 4. While designing a multiprocessor operating system, we wish to compare two different queuing schemes shown as follows.

(a) separate queues

Exponential service time

λ/2 μ

Two separate

Poisson streams

λ/2 μ

(b) common queue

λ/2 μ

Two separate

Poisson streams

λ/2 μ

Please find the average response time E[Rs] for separate queues and E[Rc] for common queues. Which one is greater?

[10%] 5. (Problem 4.3) Consider an M/Er/1 system in which *no* queue is allowed to form.

Let *j* = number of stages of service left in the system and let *Pj* be the equilibrium probability of being in state *Ej*.

* 1. Find *Pj, j = 0, 1, …, r.*
  2. Find the probability of a busy system.

[10%] 6. Starting with the Pollaczek-Khinchin equations, derive expressions for the expected waiting time *W* for an M/G/1 queue, assuming

1. M/D/1 queue with service time = 0.75 sec, and λ = 1 customer/sec.
2. Upon entry into service, a coin is tossed, which has probability *p* of giving Heads. If the result is Heads, then the service time for that customer is zero seconds. If Tails, his service time is drawn from the following exponential distribution: *pe-px*, x≧0

(mean arrival rate is λ)

1. By using M/G/1 solution formula to solve Q(z) for M/E2/1 queue.

[10%] 7. Consider an M/M/1 model with Poisson arrivals with mean rate λ and service rate μ but with the following variation: Whenever a service is completed, a departure occurs only with probability α. With probability 1 - α the customer, instead of leaving, joins the end of the queue. Note that a customer may be served more than once.

1. Set up the flow balance equations and solve for the steady-state probabilities.

State the conditions for the stationary state probabilities to exist.

1. Find the expected waiting time of a customer from the time he arrives until he gets service for the first time.