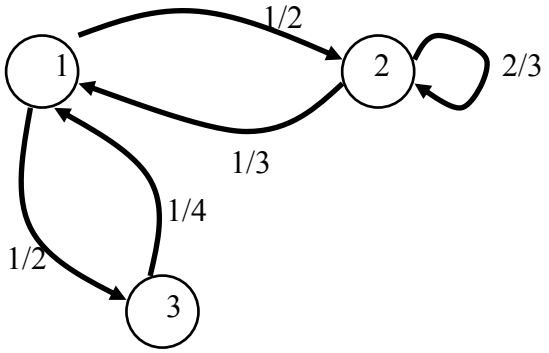
等候理論：自行練習二

1. Use Z-transforms to solve the following difference equation:

   $2g_n - 5\,g_{n-1} + 2g_{n-2} = 5(1/3)^n \qquad n = 2, 3, 4, \ldots \qquad given\ g_0=0,\ g_1=4/3$

2. A miner is trapped in a mine containing three doors. The first door leads to a tunnel that takes him to safety after two hours of travel. The second door leads to a tunnel that returns him to the same mine after three hours of travel. The third door leads to a tunnel that returns him to the same mine after five hours of travel. Assuming that the miner is at all times equally likely to choose any one of the doors, compute the probability generating function of X, the time when the miner reaches safety. Using the probability generating function, compute the mean and variance of X.
3. Jobs arriving to a computer system have been found to require CPU time that can be modelled by an exponential distribution with parameter 1/140 per millisecond. The CPU scheduling discipline is quantum-oriented so that a job not completing within a quantum of 100 milliseconds will be routed back to the tail of the queue of waiting jobs. Find the probability that an arriving job will be forced to wait for a second quantum. Of the 800 jobs coming in during a day, how many are expected to finish within the first quantum?
4. Gamma particles hit a chip in the computer of the space shuttle according to a Poisson process with rate λ. Each time the chip is hit there is a probability p that the chip is damaged and becomes inoperable.
   Let T be the length of the mission time of the space shuttle. In terms of λ, p, and T, what is the probability that the chip is operational throughout the entire mission?
5. Assuming the average number of new students entering the NCCU is 1200 every year. The average time that a student spends in NCCU is 4.5 years. What is the average number of students in NCCU every year?
6. Consider the homogeneous Markov Chain whose state diagram is shown below. Find P, the state transition matrix.



7. Consider a stack being manipulated in a program. Suppose there are only three kinds of instruction in the program, that is, push, pop and access (access operation does not change the size of the stack). We are interested only in the size of the stack, if the number of elements in the stack is i, then we say that the program is in state s_i. Let the probabilities associated with the next operation on the stack be given by:

   b = P("next operation is a push operation"),
   d = P("next operation is a pop operation"),
   a = P("next operation is an access operation"),

   and b + d + a = 1. Assume the maximal capacity of the stack is m. We say the stack has overflowed if its size is m, and the next operation is push, and has underflowed if its size is 0, and the next operation is pop. The operations causing overflow or underflowed have no effect, i.e., the stack is not changed and the program continues as before.

   (a) Draw the state diagram and find the steady state probabilities.

   (b) What is the probability that a pop operation causes an underflow?