

#### Public-key Infrastructure (PKI)

- The biggest challenge in PKC is ensuring the authenticity of public keys
- Certificates is used to help authenticate public keys6/12/2008
- PKI is a secure system that is used to manage and control certificates

- A PKI is the basis of a pervasive security infrastructure whose services are implemented and delivered using publickey concepts and techniques
- It is an infrastructure
- It is a software on users' computers
- Users might not even be aware of the PKIrelated procedures

#### Components of PKI

- Certificate issuance
- Certificate revocation
- Key backup/recovery/update
- Time stamping
- Secure communication
- Access control
- ...

#### A practical Protocol : Secure Socket Layer (SSL)

An SSL session is used to facilitate online purchases from a company's web page using a web browser





- $K_1$  is used to authenticate date using MAC •  $K_2$  is used to encrypt and decrypt data
- Only the serve is required to supply a certificate
- The client may not even have a pk

#### Certificates

- A certificate binds an ID to a public key
- Using done by having a trusted authority (a certification authority, CA) sign the information on a certificate
- Everyone can access to the PK of the CA

#### **PKI Trust Models**

- Hierarch Model
- Networked PKIs
- The Web Browser Model
- PGP

#### **Hierarchy Model**

- The root CA has a self-signed, self-issued certificate
- The root CA issues certificates for lowerlevel CAs
- Any CA can issue certificates for end users





 $x \rightarrow y$ : X signed a certificate for y

#### Verification

- Alice verifies Bob
  - Bob send all the certificates in the path  $CA_{root} \rightarrow CA_1 \rightarrow Bob$
- Alice validates Cert(CA<sub>root</sub>) using the key ver<sub>CA-root</sub>
- Alice validates Cert(CA<sub>1</sub>) using the key ver<sub>CA-root</sub>
- Alice extracts the key ver<sub>CA-1</sub> from Cert(CA<sub>1</sub>)
- Alice validates Cert(Bob) using the key ver<sub>CA-1</sub>
- Alice extracts Bob's public key from Cert(Bob)

#### **Networked PKIs**

#### Connect root CAs of two or more different PKI domains by cross-certification



#### The Web Browser Model

- Most web browsers (e.g., Netscape of Internet Explorer) come preconfigured with a set of independent root CAs
- All of which are treated by the user of browse as trust CAs
- 🗣 Ex. SSL

#### **PGP Model**

- A friend's friend is my friend
- Every user is his own CA
- Certificate Chain
  - if Alice trust Bob
  - if Bob trust Cindy
  - if I trust Alice, than
  - I trust Bob and Cindy
- It works best within a local community where most users know each other







- 密碼是保護有價物品的技術的總稱,即在保護資訊 的完整性(integrity)與私密性(confidentiality)
- 密鑰管理方式的安全與否,關係到整個密碼系統的安全性
- 秘密分享機制即為一個安全及有效的密鑰管理方案

#### Secret Splitting

## Secret Splitting (There are ways to take a message and divide it up into pieces.)

Dealer can split a message between Alice and Bob:

- Dealer generates a random-bit string, R, the same length as the message, M.
- Dealer XORs M with R to generate  $S = M \oplus R$
- Dealer gives R to Alice and S to Bob.
- (To reconstruct the message, Alice and Bob have only one step to do as following:)
- Alice and Bob XOR their pieces together to reconstruct the message  $M = R \oplus S$ .

### Dealer divides up a message into n pieces

- Dealer divides up a message into n pieces among U<sub>1</sub>, ..., and U<sub>n</sub>.
- Dealer generates n-1 random-bit strings, R<sub>1</sub>, ..., and R<sub>n-1</sub>, the same length as the message, M.
- Dealer XORs M with the n-1 strings to generate  $R = M \oplus R_1 \oplus ... \oplus R_{n-1}$ .
- Dealer gives  $R_1$  to  $U_1$ , ...,  $R_{n-1}$  to  $U_{n-1}$ , and R to  $U_n$ .



- (U<sub>1</sub>, ..., and U<sub>n</sub>, working together, can reconstruct the message as following.)
   U<sub>1</sub>, ..., and U<sub>n</sub> get together and compute:
- $\mathbf{M} = \mathbf{R} \oplus \mathbf{R}_1 \oplus \ldots \oplus \mathbf{R}_{n-1}.$



#### **Real Applications**

## In USB TokensIn PGP



#### What is Secret Sharing



- If we want the secret file to be opened only when six or more then six of the scientists are to participate, then,
- 1. How many locks shall we install on the secret file at least?
- 2. How many keys should every scientist have at least?
- 1:  $C_6^{11} = 462..$
- 2: C<sup>10</sup> = 252.

#### **Real Applications**

## In banksIn military (missile launch system)

(*k*, *n*)-門檻秘密分享方案 (*k*, *n*)-threshold scheme

- A (*k*, *n*) threshold scheme is a method of breaking a secret  $\bigotimes$  up into *n* different shares,  $S_1, \ldots, S_n$ , such that:
  - With the knowledge of any k or more shares  $(k \le n)$ ,

the secret  $\bigotimes$  can be easily derived; and



#### (k, n)-threshold scheme

• A (k, n) threshold scheme is a method of breaking a secret  $\aleph$  up into *n* different shares,  $S_1, \ldots, S_n$ , such that:

With the knowledge of any k-1 or fewer shares, it is impossible to derive the secret  $\aleph$ .



#### Shamir's (*k*,*n*)-threshold scheme\*

• 
$$f(x) = a_0 + a_1 x + \dots + a_{k-1} x^{k-1} \in Zp[x]$$

- The secret :  $f(0) = a_0$
- The set of *n* shares:  $S = \{(x_i, y_i) : 1 \le i \le n, y_i = f(x_i)\}$
- By using the Lagrange interpolation formula, any k of the n participants pooling their shares can easily reconstruct the secret.

$$f(0) = a_0 = \sum_{i=1}^k y_i \times \prod_{\substack{1 \le l \le k \\ l \ne 1}} \frac{x_l}{x_l - x_i}$$

\*A. Shamir, "How to share a secret", Comm. ACM 22 (1979), 612-613.

#### Shamir's (*3*,*n*)-threshold scheme

- The secret : S = f(0)
- The set of *n* shares:  $(x_i, y_i), 1 \le i \le n$



#### The Shamir Threshold Scheme

Let t, w be positive integers, t ≤ w. A (t, w) threshold scheme is a method of sharing a key K among a set of w participants (denoted by ρ), in such a way that any t participants can compute the value of K, but no group of t-1 participants can do so.

#### Shamir (t, w)-Threshold Scheme

# There are two phases: Initialization Phase Share Distrdibution Phase

#### **Initialization Phase**

- D choose w distinct, non-zero elements of  $Z_p$ , denoted  $x_i$ .  $1 \le i \le w$  (this is where we require  $p \ge w+1$ ).
- For  $1 \leq i \leq w$ , D gives the value  $x_i$  to  $P_i$
- The values x<sub>i</sub> are public.

#### **Share Distribution**

- Suppose D wants to share a key  $K \in Z_p$ .
- D secretly chooses (indepently at random) t-1 elements of Z<sub>p</sub>, which are denoted a<sub>1</sub>, ..., a<sub>t-1</sub>.
- For  $1 \le i \le w$ , D computes  $y_i = a(x_i)$ , where  $a(x) = K + \sum_{j=1}^{t-1} a_j x^j \mod p$ .
- For  $1 \leq i \leq w$ , D gives the share  $y_i$  to  $P_i$ .

#### Description

- K: secret.
- Polynomial:  $\mathbf{a}(\mathbf{x}) = \mathbf{K} + \sum_{j=1}^{T-1} a_j \mathbf{x}^j$
- $\rho = \{Pj: 1 \leq j \leq w\}$ : the set of w participants.
- Every participant P<sub>j</sub> obtains a point (x<sub>j</sub>, y<sub>j</sub>) on this polynomial.
- Suppose that participants P1, ..., Pt want to determine K. They know that y<sub>i</sub> = a(x<sub>i</sub>), 1≤ j ≤ t.

#### Example

- Suppose that p = 17, t = 3, and w = 5; and the public x-coordinates are x<sub>i</sub> = j, 1 ≤ j ≤ 5.
- Suppose that B = { P<sub>1</sub>, P<sub>3</sub>, P<sub>5</sub>} pool their shares, which are respectively 8, 10, 11.
- Write the polynomial  $a(x) = a_0 + a_1x + a_2x^2$ , and computing a(1), a(3),  $a(5) \in \mathbb{Z}_{17}$ .
- We will have  $(a_0, a_1, a_2) = (13, 10, 2)$ .
- The key is  $K = a_0 = 13$ .

#### Lagrange Interpolating Formula

- Given  $(x_j, y_j)$ ,  $1 \le j \le t$ , there exists a polynomial a(x) so that  $y_j = a(x_j)$ .
- The formula for a(x) is as follows:

$$a(x) = \sum_{i=1}^{t} y_i \prod_{j=1, j \neq i}^{t} \frac{x - x_j}{x_i - x_j} \mod(p).$$



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 The formula can be expressed as follows: a(x) = a<sub>0</sub> + a<sub>1</sub>x + a<sub>2</sub>x<sup>2</sup> + ... + a<sub>t-1</sub>x<sup>t-1</sup>.
 The key K = a<sub>0</sub> = a(0).
 K = a(x) = ∑\_{i=1}^{t} y\_i \prod\_{j=1, j \neq i}^{t} \frac{x\_j}{x\_i - x\_j} \mod(p).

#### Simplified (t, t)-Threshold Scheme

- D secretly chooses (independently at random) t-1 elements of Z<sub>m</sub>, y<sub>1</sub>, ..., y<sub>t-1</sub>.
   D computer w = K × ∑<sup>t-1</sup> w mod m
- **D** computes  $\mathbf{y}_t = \mathbf{K} \sum_{j=1}^{t-1} \mathbf{y}_j \mod \mathbf{m}$ .
- For  $1 \leq j \leq t$ , D gives the share  $y_j$  to  $P_j$ .

#### Description

## • Observe that the t participants can compute K by the formula $\mathbf{K} = \sum_{j=1}^{t} \mathbf{y_j} \mod \mathbf{m}$ .

#### Example – Finding Key

- Suppose that m = 10 and t = 4.
- Suppose that the shares for the four participants are  $y_1 = 7$ ,  $y_2 = 2$ ,  $y_3 = 4$  and  $y_4 = 2$ .
- The key  $K = 7 + 2 + 4 + 2 \mod 10 = 5$ .

#### Example -- Security

- Suppose that the first three participants try to determine K.
- They know that y<sub>1</sub> + y<sub>2</sub> + y<sub>4</sub> mod m = 3, but they do not know the value y4.
- There is a one-to-one correspondence between the ten possible values of y<sub>4</sub> and the ten possible values of the key K.





#### 視覺型秘密分享機制



42











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#### 政大資科碩專班

47







- 公開金鑰密碼的安全性建立在許多假設 性的難問題上(DL-Problem,CDH Problem, Prime Factoring)
- 針對這些難問題,如果發現新演算法, 則安全性有可能一夕之間瓦解
- 量子電腦(Quantum)

量子密碼

## 絕對安全的密碼(終極密碼) 終結了編碼者與解碼者的戰爭

#### 量子的四種狀態(|-/\)



53

#### 測不準原理



